International Journal of Solids and Structures 50 (2013) 833–842

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Three-dimensional boundary element formulation of an incompressible viscoelastic layer of finite thickness applied to the rolling resistance of a rigid sphere

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ARTICLE INFO

Article history:
Received 30 April 2012
Received in revised form 12 November 2012
Available online 5 December 2012

Keywords:
Rolling resistance
Boundary element method
Viscoelastic layer
Hysteretic friction
Coulomb's friction
Fourier series

ABSTRACT

A three-dimensional boundary element formulation of an incompressible viscoelastic layer of finite thickness is proposed, in a moving frame of reference. The formulation is based on two-dimensional Fourier series expansions of relevant mechanical fields in the continuum of the layer. The linear viscoelastic material is characterized, in the most general way, by its frequency-domain master curves. The presented methodology results in a compliance matrix for the layer's upper boundary, which includes the effects of steady-state motion and can be used in any contact problem-solving strategy. The proposed formulation is used, in combination with a contact solver, to build a full three-dimensional model for the steady-state rolling/sliding resistance incurred by a rigid sphere on the layer. Energy losses include viscoelastic damping and surface friction. The model is tested and its results are found to be consistent with existing solutions in limiting cases. An example is explored and the corresponding results are used to illustrate the influence of different parameters on the rolling resistance. General aspects of previously-described dependences are confirmed.

1. Introduction

For diverse reasons, rolling resistance remains important to many engineering applications. From nanotechnologies and molecular dynamics (e.g. Lee et al., 2009) to various scale industrial applications and transportation purposes (e.g. Hall, 2001; Qiu, 2006; Qiu, 2009), from earthquake hazards mitigation, to energy harvesting and sustainable development considerations (e.g. Sharp, 2009), depending on human's objectives and goals, rolling resistance may be fiercely avoided or eagerly sought and thus requires careful attention.

Rolling resistance has been, and still is, widely addressed in scientific literature. In 1785 experiments on friction were reported by Coulomb (1821) and Vince and Shepherd (1785). Further experiments led to significant progress during the 1950's and the early 1960's towards a better understanding of its complex nature, involving surface contact phenomena as well as bulk properties of the interacting materials (Greenwood et al., 1961; Tabor, 1955). Hysteretic friction in the bulk is revealed in many works on nonstationary viscoelastic contact problems, in various settings (e.g. Barber et al., 2008; Chertok et al., 2001; Galin and Gladwell, 2008; Golden and Graham, 2001; Morland, 1967; Morland, 1968; Wang and Knothe, 1993). In particular, rolling friction of hard cylinders was approached in two dimensions using various methods (e.g. Hunter, 1961; Johnson, 1985; May et al., 1959; Morland, 1967; Põschel et al., 1999) and its dependence upon physical parameters was modeled based on simplifying assumptions regarding the description of the foundation layer and/or the nature of contact interactions. A one-dimensional treatment of a hard sphere rolling on a viscoelastic half-space modeled using a 'Winkler' approximation was given by Flom and Bueche (1959). In the absence of surface friction, a "first-principle" (i.e. free of empirical parameters) continuum-mechanics expression of the rolling resistance coefficient was derived by Brilliantov and Pőschel (1998) for the rolling motion of a viscoelastic sphere on a hard plane, in quasi-static conditions, such that the total stress field may be considered as the sum of an elastic part and a dissipative part, and the vertical displacement field may be approximated by the corresponding result of the static problem.

More recently, numerical difficulties associated with enforcing frictional conditions on finite element models of hyperelastic tires rolling in steady state conditions on rigid surfaces, were tackled by Laursen and Stanciulescu (2006) and Stanciulescu and Laursen (2006). A full two-dimensional boundary element formulation for a hard cylinder rolling on a viscoelastic layer of finite thickness was introduced by Qiu (2006) while Persson (2010) presented an approach to calculate the rolling resistance of hard objects on viscoelastic solids using a static pressure distribution. Alternative approaches to estimating the viscoelastic rolling resistance on a sphere in 3D are presented by Zéhil and Gavin (2012). More comprehensive solutions to the problem of rolling resistance in three

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http://dx.doi.org/10.1016/j.ijsolstr.2012.11.020

dimensions, including frictional effects, remain however in need. Furthermore, the increasing complexity of numerical models requires investigating possible ways of reducing their computational costs and hence improving their efficiency.

In this paper, we present a three-dimensional boundary element formulation of an incompressible linear viscoelastic layer of finite thickness, in a moving frame of reference. This formulation is applied, in combination with a contact solver, to build a full three-dimensional model for the resistance incurred by a rigid object (sphere) rolling/sliding on the layer, including surface friction. Inspired by the seminal work of Qiu (2006), we expand relevant mechanical fields in the continuum of the layer into two-dimensional Fourier series. The storage and loss moduli characterizing the constitutive behavior of linear viscoelastic materials, in the frequency domain, are used to relate the Fourier coefficients. The proposed formulation results in the assembly of a compliance matrix **C** characterizing the behavior of the layer's upper boundary, including the effects of steady-state motion. This compliance matrix may be used in any stationary or steady-state rolling/sliding contact problem-solving strategy. The proposed formulation is quite general and practical in that it accommodates any linear viscoelastic model, including experimental master-curves. In order to increase its computational efficiency, special attention is given to exploiting configurational similarities as well as symmetry.

2. Defining rolling resistance

Fig. 1 shows a round rigid object (cylinder or sphere of center C and radius R) rolling in steady-state conditions, on a viscoelastic layer of finite thickness H. The object moves in direction x at a constant linear velocity V_s while rotating about its axis at a rotational speed Ω. It is subjected to a vertical load P (positive downwards), a driving horizontal force Q (positive in the direction of increasing x) and a driving torque T (positive clockwise). The indentation d corresponds to the maximum penetration of the rolling object below the surface of the unloaded layer.

Because the contact surface takes the form of the rigid object, tangential shear stresses are circumferential and normal stresses are radial, with respect to a polar coordinate system centered at point C. However, contact stresses can be re-expressed in the Cartesian coordinate system $Oxyz$ as well.

For the purposes of this work, rolling resistance is defined as a conceptual horizontal resisting force R_r, expressed as a positive quantity. If it were to be applied at the axis of the moving object, the rolling resistance would dissipate energy at a rate that is equivalent to the power dissipation actually incurred by the system. Rolling resistance R_r is related to Q and T by

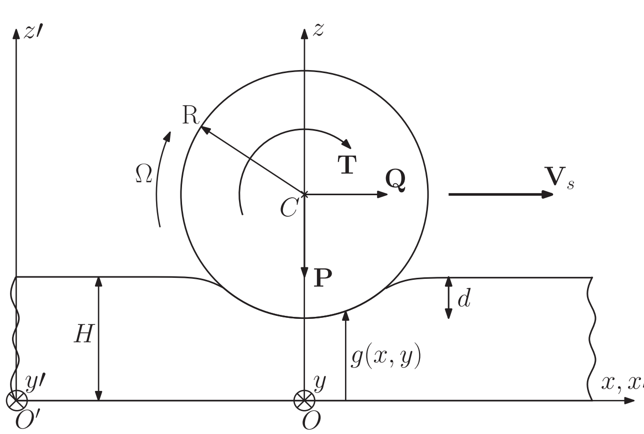

Fig. 1. General model and coordinate systems.

$$R_r = Q + \frac{T\Omega}{V_s}. \tag{1}$$

In Eq. (1), R_r is the rolling resistance corresponding to the total dissipated power. It is considered here that, in the absence of surface friction, rolling resistance is entirely due to viscous dissipations in the bulk. In such cases, the value of a driving torque is indeterminate, as it can not be equilibrated, and rolling resistance is equal to the sum of the horizontal projection of the radial contact forces. Because of the asymmetry of these forces, the rolling resistance is non-zero.

In the presence of friction, the interfacial shear stresses are not zero and a driving torque T can be balanced by either taking the moment of the tangential contact forces about the roller axis, or taking moments of the vertical and horizontal components of the contact forces about the same axis. Surface frictions influence rolling resistance in two ways: (i) directly, by means of their resisting work localized in the slipping regions of the contact surface, and (ii) indirectly, as demonstrated by Munisamy et al. (1991), by modifying the (frictionless) contact pressure distribution, which further impacts the global energy balance. The contribution of slipping friction to rolling resistance may be evaluated as follows

$$R_r^F = \frac{1}{V_s}\int_{A_c} \mathbf{w}_t.\boldsymbol{\tau}_t dA, \tag{2}$$

where A_c stands for the contact area, $\mathbf{w}_t$ is the local tangent differential speed between the sphere and the foundation layer and $\boldsymbol{\tau}_t$ corresponds to the tangent stress field across the contact interface. In the presence of friction, the rolling resistance attributed to the viscoelastic behavior of the layer, is obtained by subtraction

$$R_r^V = R_r - R_r^F. \tag{3}$$

A common case is when the horizontal driving force Q is applied at the top of the moving object, thus generating a dependent torque $T = QR$. Substituting into expression (1) yields

$$R_r = Q\left(1 + \frac{R\Omega}{V_s}\right). \tag{4}$$

3. Governing equations

Following the development of Qiu (2006), the viscoelastic layer of thickness H is assumed to be incompressible, sustains small deformations and behaves linearly. As shown in Fig. 1, $Oxyz$ corresponds to a moving coordinate system traveling with the sphere, while $O'x'y'z'$ remains at rest. Both coordinate systems are related according to

$$x = x' - V_s t, \quad y = y', \quad \text{and} \quad z = z'. \tag{5}$$

Also, in the traveling coordinate system, material derivatives are expressed such that time becomes an implicit variable

$$\frac{D}{Dt} = -V_s\frac{\partial}{\partial x}; \quad \frac{D^2}{Dt^2} = V_s^2\frac{\partial^2}{\partial x^2}. \tag{6}$$

The equilibrium equations for the elastomer in $O'x'y'z'$ are given, in tensorial form, by

$$\rho\frac{D^2\mathbf{u}}{Dt^2} = \mathbf{div}'(\mathbf{s}) - \mathbf{grad}'(p), \tag{7}$$

where $\mathbf{u} = <u, v, w>^T$ is the displacement field, ρ stands for the material's density, p is the pressure and $\mathbf{s}$ denotes the stress deviator. Eq. (7) may be expressed in $Oxyz$ using (6) and hence becomes

$$\rho V_s^2\frac{\partial^2\mathbf{u}}{\partial x^2} = \mathbf{div}(\mathbf{s}) - \mathbf{grad}(p). \tag{8}$$