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Simplified approaches to viscoelastic rolling resistance

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ABSTRACT

Modeling approaches yielding rolling resistance estimates for rigid spheres (and cylinders) on viscoelastic layers of finite thicknesses are introduced as lower-cost alternatives to more comprehensive solution-finding strategies. Detailed examples are provided to illustrate the capabilities of the different approaches over their respective ranges of validity.

1. Content summary

This paper clearly delineates two approaches to estimating the rolling resistance of a rigid sphere on a viscoelastic layer of finite thickness. In both approaches, the effects of slipping friction are neglected.

In the 2D Cylinder-based approach, the sphere is conceptually sliced into a set of cylinders. The rolling resistance incurred by each cylinder is determined by solving a rolling contact problem in two dimensions. The foundation's behavior is described by a numerical compliance in plane strain. The 2D cylinder-based approach builds on results from Qiu (2006) and involves new approximate methods of extending two-dimensional models of rolling cylinders to modeling a rolling sphere in three dimensions. Three numerical methods of varying complexity and accuracy are presented for this approach.

In the Direct Fourier series approach, rolling resistance is estimated by computing dissipated power, in the vertical direction, along the contact surface. This approach mainly relies upon the approximate assumption that the stationary vertical stress distribution, as well as the corresponding contact area, are unaltered by motion. Inspired by the recent work of Persson (2010) on rolling resistance, and building upon results from Zéhil and Gavin (submitted for publication-a,b), as well as on stationary contact results from Jaffar (1988, 1997, 2008), new expressions for the rolling resistance are derived, for different ranges of foundation thickness, in the form of direct Fourier expansions.

2. Introduction and motivation

A full three-dimensional model of a rigid sphere, rolling in steady-state, with or without friction, on a viscoelastic foundation of finite thickness is presented in Zéhil and Gavin (submitted for publication-a). The candidate contact surface is discretized in a coordinate system that is traveling along with the moving object and the foundation's behavior described using a three-dimensional boundary element formulation, yielding a constitutive model for the layer of the following form

$$\mathbf{CF} = \mathbf{D} \tag{1}$$

where $\mathbf{F}$ is a nodal surface force vector, $\mathbf{D}$ the corresponding nodal surface displacement vector and $\mathbf{C}$ the foundation's compliance matrix. Full results are then obtained by solving the rolling contact problem at the interface between the rigid sphere and the viscoelastic layer described by Eq. (1). Efficient means of solving the rolling contact problem are described in Zéhil and Gavin (submitted for publication-b).

The practical implementation of the full three-dimensional model involves determining matrix $\mathbf{C}$, or at least relevant parts of it, depending on each problem's particular assumptions and goals. Assuming that the candidate contact surface is discretized into K_x and K_y nodes in directions x and y respectively (see Fig. 1), the total number of nodes is $N_T = K_xK_y$ and the full resulting square compliance matrix $\mathbf{C}$ is of dimension $3N_T = 3K_xK_y$. Taking advantage of existing configurational similarities between pairs of nodes, less

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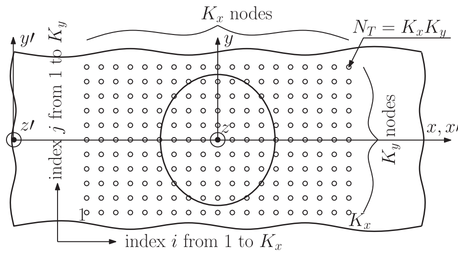

Fig. 1. Discretization of the candidate contact surface.

than six columns of **C** (i.e. about $18K_xK_y$ terms) need to be formed explicitly (see Zéhil and Gavin, submitted for publication-b).

In the absence of friction and provided that no horizontal displacements are wanted as part of the solution, only 1/9th of the matrix **C** is required, from which less than two columns need to be formed explicitly, which adds up to about $2K_xK_y$ terms. This number remains relatively high considering the fact that each matrix entry results from the addition of a sufficient number of terms from a double Fourier series. The computational cost of building compliance matrices increases quadratically with the number of discretization nodes and the truncation order. For reference, in the absence of surface friction, the computational time of building 1/9th of a 3D compliance matrix corresponding to $N_T = 41 \times 41 = 1681$ nodes and including $N_{tx} = N_{ty} = 500$ Fourier terms on an Intel® Core™ i7 M620 CPU with 4 MB of cache memory and a clock speed of 2.66 GHz is approximately two minutes. In comparison, solving a frictionless rolling contact problem in 3D, using the same hardware, requires roughly 1.33 s. Consequently, the total computational time needed to evaluate the rolling resistance for 23 different values of rolling speed V_s and 15 different values of the applied load P, as we do in Sections 3.4 and 4.6 of this manuscript, adds up to almost 54 min. This is considered as the reference case.

In many circumstances however, a complete and perfectly accurate solution is not necessary, hence justifying the search for cheaper computational means. This is particularly the case when only an estimate of the rolling resistance is sought. The present work considers alternative approaches to the full three-dimensional model, providing rolling resistance estimates with an accuracy that is suitable for many engineering purposes. According to Qiu (2006) and Zéhil and Gavin (submitted for publication-a), the contribution of surface friction to the total rolling resistance is relatively small in comparison with viscoelastic energy dissipation and will therefore be neglected. Experimental evidence strongly supporting this assumption, for the rolling and lubricated sliding of rigid cylinders and spheres on rubber, date back to the 1950s (e.g. Greenwood and Tabor, 1958; Tabor, 1955).

3. 2D Cylinder based approaches to a 3D rolling resistance problem

Although configurational similarities afford significant increases of efficiency in three-dimensional solutions, the computational cost remains high in comparison with a similar implementation of a two-dimensional model originally presented by Qiu (2006). We have thus sought approximate solutions for the rolling resistance on a sphere (which is a 3D problem) based on a two-dimensional model of a rolling cylinder.

3.1. Shared principle

The idea is quite simple: a sphere of radius R moving in direction x at a given speed V_s, as depicted in Fig. 2, is conceptually divided, along the transverse direction y, into an odd number of thin vertical cylindrical elements of thickness dy, such that one slice is centered in the middle with a symmetrical discretization on both sides. A cylindrical slice whose middle layer is centered at y has radius $R_c(y)$ and penetration $d_c(y)$. In particular, the middle slice is centered at $y = 0$ and its radius $R_c(0)$ is equal to R. The behavior of each cylindrical element is then approached using a two-dimensional model where the underlying subbase is in a state of plane strain (which is where the approximation lies). The vertical load P that is applied to the sphere gets distributed among the cylindrical slices. At equal thickness, the middle slice supports the largest part of the load and thus incurs the largest penetration $d_c(0)$. However, due to the plane stain assumption that is made on the layer in two-dimensions, $d_c(0)$ is typically smaller than the actual penetration of the sphere (d), as determined from a 3D model.

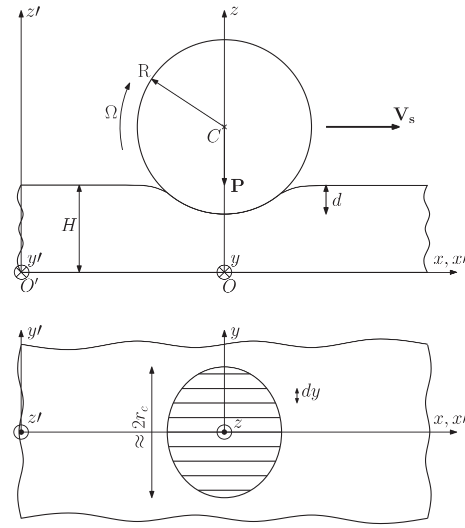

Fig. 2. Discretization of the sphere into cylindrical elements (section in plane *Oxz* and projection on plane *Oxy*).

The total rolling resistance on the sphere is estimated by summing the rolling resistances incurred by each of its cylindrical elements, taking advantage of symmetry. Three variant algorithms (named "PD", "PP" and "SP") based on these common principles were implemented and tested against 3D results. The additional assumptions specific to each algorithm are presented in the sequel.

3.2. Algorithms PD and PP

3.2.1. Common core of algorithms PD and PP

The half-width of the actual contact surface r_c is considered to be roughly equal to the contact radius of a perfectly centered and circular one

$$r_c = \sqrt{d_o(2R - d_o)} \tag{2}$$

where the penetration of the middle cylindrical slice $d_o = d_c(0)$ is approximated by the vertical distance between the bottom of the sphere and the contact boundary. It is further assumed that the marginal distribution $p_c(y)$ of the total vertical load P, among the cylindrical slices, is quadratic in y, transversally symmetric and continuous at the edges, i.e.

$$p_c(y) = \frac{r_c^2 - y^2}{2R_p} \tag{3}$$

where R_p is an unknown parameter characterizing the distribution $p_c(y)$ and corresponding to its radius of curvature, at the apex.