On the full coupling between thermo-plasticity and thermo-damage in thermodynamic modeling of dissipative materials

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ABSTRACT

The phenomenological model of dissipative material in the small strain range is developed in the framework of thermodynamics of irreversible processes with internal state variables and local state method. The possibilities of the model are illustrated in the example of thermo-elastic–plastic damage material. Particular emphasis is put on including in the description of the full coupling between plasticity and damage in nonisothermal conditions. The consequences of thermal-plastic-damage coupling in consistency conditions and loading/unloading conditions are studied in detail.

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1. Introduction

The increasing demands for high performance materials require the adequate constitutive modeling, as well as the appropriate predictions of the overall failure mechanisms under complex thermo-mechanical loads. When engineering materials classified as elastic–plastic-damage (for example polycrystalline metals) are subjected to external loading, the material degradation connected with slip rearrangements of crystallographic planes through dislocation motion, observed at the macro-scale as plastic behavior (Chaboche, 2008), is accompanied by the development of other microscopic defects, like micro-cracks and micro-voids (Lemaitre, 1992; Abu Al-Rub and Voyiadjis, 2003). The nucleation, growth and interaction of these micro-defects under external loads result in a deterioration process on the macro-scale and, as a consequence, change of the constitutive properties of the material.

If the elastic–plastic-damage material is loaded so that not only inelastic strains develop, but also the temperature is changed, then thermo-elasticity, thermo-plasticity and thermo-damage are encountered. The experimental results (Bednarek and Kamocka, 2006) proved that not only the temperature itself but also the heating rate makes a significant impact on parameters that determine carrying capacity at elevated temperatures, and that heating rate should be accounted for in the strength analysis of structures exposed to high temperatures. The need for the additional term, proportional to the temperature rate in the evolution equation for the back-stress was already considered by Prager (1958), introduced

also by Chaboche (1997b) in the unified viscoplastic constitutive equations using the Armstrong–Frederic format. In Chaboche (2008) the discussion is made for the necessity of temperature rate terms in the context of hardening rules.

Ganczarski and Skrzypek (2009) take into account the temperature dependence of all material functions that characterize plasticity and damage components, which results in extended thermo-plastic-damage equations, with the additional temperature rate terms in all evolution equations of thermodynamic conjugate forces. More general case of the non-associated plasticity and non-associated damage, when not only temperature-softening but also damage-softening is taken into account is due to Egner (2009).

In the present analysis a general phenomenological model, based on the irreversible thermodynamics, is formulated and used to describe the dissipative elastic–plastic-damage material in the small strain range. A special attention is paid to the proper description of coupling between heating rate and two dissipative phenomena: plasticity and damage, taking place in the material subjected to nonisothermal conditions. Both thermal softening and damage softening are accounted for and the consequences of coupling in consistency conditions and loading/unloading conditions are studied in detail.

2. General thermodynamical model of dissipative materials

2.1. Basic assumptions

We consider a closed thermodynamic system that is susceptible of several possibly coupled dissipative phenomena (like plasticity, damage, phase changes, frictional slips on closed crack lips etc.,)

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that are formalized on the macroscopic level by the use of a proper set of state variables. The motions of the system obey the fundamental laws of continuum mechanics (conservation of mass, conservation of linear momentum, conservation of angular momentum) and two laws of thermodynamics written here in the local form:

- Conservation of energy

$$\rho \dot{u} - \dot{\varepsilon}_{ij}\sigma_{ij} - r + q_{i,i} = 0 \tag{1}$$

- Clausius–Duhem inequality

$$\rho \dot{s} - \frac{r}{\theta} + \frac{q_{i,i}}{\theta} - q_i \frac{\theta_{,i}}{\theta^2} \geqslant 0 \tag{2}$$

where ρ is the mass density per unit volume; $\underline{\sigma}$ is the stress tensor; u is the internal energy per unit mass; $\underline{\varepsilon}$ is the strain tensor; r is the distributed heat source per unit volume; $\mathbf{q}$ is the outward heat flux; s is the internal entropy production per unit mass and θ is the absolute temperature.

Depending on the scale, different approaches may be used in order to describe an overall structural response of a dissipative structure on the macro-scale. In general, micro-mechanical models relate the macro-properties and the macro-response of a structure to its microstructure. In such approach the rearrangements of micro-structure are discrete and stochastic phenomena induced by a number of weakly or strongly interacting micro-changes that influence the overall structural response. The micro-mechanical models have the advantage of being able to sustain heterogeneous structural details on the micro-scale and meso-scale, and to allow a micro-mechanical formulation of the evolution equations based on the accurate micro-changes growth processes involved (cf. Voyiadjis et al., 2007; Boudifa et al., 2009; Aboudi, in press).

Continuum mechanics approach, applied in the present work, provides the constitutive and damage evolution equations in the framework of thermodynamics of irreversible processes. The material heterogeneity (on the micro- and meso-scale) is smeared out over the representative volume element (RVE) of the piece-wise discontinuous material. The true state of material within RVE, represented by the topology, size, orientation and number of micro-changes, is mapped to a material point of the quasi-continuum. The true distribution of micro-changes within the RVE, and the correlation between them are measured by the change of the effective constitutive properties. The micro-structural rearrangements are defined by the set of state variables of the scalar, vectorial or tensorial nature (cf. Murakami and Ohno, 1980; Chaboche, 1997a; Skrzypek et al., 2008; Ganczarski et al., 2010).

In the case of infinitesimal deformation the total strain $\underline{\varepsilon}$ can be expressed as the sum of the elastic (reversible) strain $\underline{\varepsilon}^E$, inelastic (irreversible) strain $\underline{\varepsilon}^I$, and thermal strain $\underline{\varepsilon}^\theta$:

$$\varepsilon_{ij} = \varepsilon_{ij}^E + \varepsilon_{ij}^I + \varepsilon_{ij}^\theta \tag{3}$$

In the process of deformation, various microstructural rearrangements of material structure may be induced, for example the changes in density and configuration of dislocations, the development of microscopic cavities, changes from primary to secondary phase etc. All these rearrangements may contribute to both reversible and irreversible strains (cf. Abu Al-Rub and Voyiadjis, 2003), therefore:

$$\varepsilon_{ij}^E = \varepsilon_{ij}^e + \sum_{k=1}^{n} \varepsilon_{ij}^{Ek}, \quad \varepsilon_{ij}^I = \sum_{k=1}^{n} \varepsilon_{ij}^{Ik}, \quad \varepsilon_{ij}^k = \varepsilon_{ij}^{Ek} + \varepsilon_{ij}^{Ik}, \quad k = 1, 2, \ldots, n \tag{4}$$

where ε_{ij}^e is a "pure" elastic strain, and $\varepsilon_{ij}^{Ek}, \varepsilon_{ij}^{Ik}$ are respectively the reversible and irreversible components of the total strain ε_{ij}^k induced by k-th dissipative phenomenon (see Fig. 1), e.g. plastic flow ($k = p$), damage ($k = d$), phase change ($k = p_h$) etc. For example, in the case of thermo-elastic–plastic-damage material the total strain tensor is expressed as

$$\varepsilon_{ij} = \underbrace{\varepsilon_{ij}^e + \varepsilon_{ij}^{Ed}}_{\varepsilon_{ij}^E} + \underbrace{\varepsilon_{ij}^p + \varepsilon_{ij}^{Id}}_{\varepsilon_{ij}^I} + \varepsilon_{ij}^\theta \tag{5}$$

while its damage induced component, ε_{ij}^d, consists of both reversible (Ed) and irreversible (Id) damage strain terms

$$\varepsilon_{ij}^d = \varepsilon_{ij}^{Ed} + \varepsilon_{ij}^{Id} \tag{6}$$

2.2. State variables

The irreversible rearrangements of the internal structure can be represented by a group of state variables describing the current state of material microstructure:

$$\{\Lambda^k\}, \quad k = p, d, ph, \ldots \tag{7}$$

where Λ^k may be scalars, vectors or even rank tensors. For damage description, in the case where the damaged material remains isotropic, the current state of damage is often represented by the scalar variable Λ^d denoting the volume fraction of cracks and voids in the total volume.

Damage acquired orthotropy requires a second order tensor, for example the classical (Murakami and Ohno, 1980) tensor:

$$\underline{\Lambda}^d = \underline{\mathbf{D}} = \sum_{i=1}^{3} D_i n_i \otimes n_i \tag{8}$$

where $D_i = dA_i^d/dA_i$ denotes the ratio of cracks and voids area to the total area on the principal plane of normal unit vector $\underline{\mathbf{n}}$. In the most general case of anisotropy the description of damage needs to be embodied in an eight-order tensor (cf. Cauvin and Testa, 1999), while the principle of strain equivalence allows using fourth-order tensors.

For the phase transformation analysis the scalar variable

$$\Lambda^{ph} = \xi = \frac{dV^s}{dV^0} \tag{9}$$

is commonly adopted (cf. Egner and Skoczeń, 2010), which denotes the volume fraction V^s of the martensite in the total volume V^0 of the martensite-austenite representative volume element, if martensitic transformation $\gamma \to \alpha'$ is considered. However, a scalar variable is not capable of describing the acquired anisotropy due to partially directional nature of the martensitic inclusions in the austenitic matrix. Therefore, instead of scalar variable (9) a second-order tensor can be defined in analogy to (8):
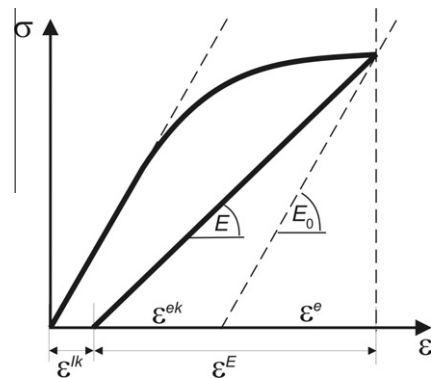

Fig. 1. Components of the strain tensor induced by k-th dissipative phenomenon.