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Finding minimum energy configurations for constrained beam buckling problems using the Viterbi algorithm

Srikrishna Doraiswamy, Krishna R. Narayanan, Arun R. Srinivasa *

Texas A&M University, College Station, TX 77843, USA

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ABSTRACT

In this work, we present a novel technique to find approximate minimum energy configurations for thin elastic bodies using an instance of dynamic programming called the Viterbi algorithm. This method can be used to find approximate solutions for large deformation constrained buckling problems as well as problems where the strain energy function is non-convex. The approach does not require any gradient computations and could be considered a direct search method. The key idea is to consider a discretized version of the set of all possible configurations and use a computationally efficient search technique to find the minimum energy configuration. We illustrate the application of this method to a laterally constrained beam buckling problem where the presence of unilateral constraints together with the non-convexity of the energy function poses challenges for conventional schemes. The method can also be used as a means for generating "very good" starting points for other conventional gradient search algorithms. These uses, along with comparisons with a direct application of a gradient search and simulated annealing, are demonstrated in this work.

1. Introduction

The aim of this work is to demonstrate a new technique based on dynamic programming (Bellman, 1954) to obtain minimum energy configurations for certain types of elasticity problems. Though instances of dynamic programming have been used for various purposes, hitherto such an approach has not been exploited in elasticity, because dynamic programming is generally used as a technique to solve certain kinds of discrete problems. The extension of dynamic programming to problems in elasticity enables a new perspective to solve these problems. A specialized version of dynamic programming called the Viterbi algorithm (Viterbi, 1967) is utilized here. This algorithm has been used to find global minima of a variety of cost functions in a number of applications such as in communication theory (Forney, 1973), computer vision (Oliver et al., 2000) and peptide sequencing (Fischer et al., 2005) etc. This work introduces the application of Viterbi algorithm to problems with unilateral constraints in elasticity.

To explain the working of the algorithm, an example problem of finding the minimum energy configuration of a cantilever beam under the action of compressive as well as lateral tip loads, confined to deform within two lateral walls (see Fig. 1) is solved in this work. This problem is chosen because it is relatively easy to evaluate the performance of the algorithm. The problem is first discretized by assuming the beam to be a connection of discrete links. To find the global minimum, we search through the configurations of the beam. This search can obviously only be performed on finite sets. So the space of possible configurations is discretized, also known in this paper as *range discretization*. The main idea behind the algorithm is to consider this finite subset of all possible configurations of the beam and cleverly search through to find the lowest energy configuration in this discretized space.

Conventional gradient descent methods for minimization perform extremely well for smooth and/or convex functions and/or when the initial guess is close to a local minimum. In the case of heuristic techniques (Yang, 2010; Kirkpatrick et al., 1983) where the results are heavily dependent on the value of the parameters involved and choice of the initial guess,[1] the solution is not guaranteed to be the global minimum. On the other hand for problems where the energy is non-convex/non-smooth and where there are constraints such as the cantilever beam problem considered here, these techniques do not perform well unless one is close to a minimum. For example, gradient search techniques give quadratic convergence rates when one is close to a minimum, but there is no a priori way to find initial guesses which are close to minima. The algorithm presented in this work can be used in two ways: a stand-alone technique for finding the global minimum for the discretized

* Corresponding author.
E-mail addresses: srikrishna@tamu.edu (S. Doraiswamy), krn@ece.tamu.edu (K.R. Narayanan), asrinivasa@tamu.edu (A.R. Srinivasa).

[1] Such methods were indeed implemented for the problem stated in Section 2 and the dependence of the solution on the parameters is explained in Section 5.

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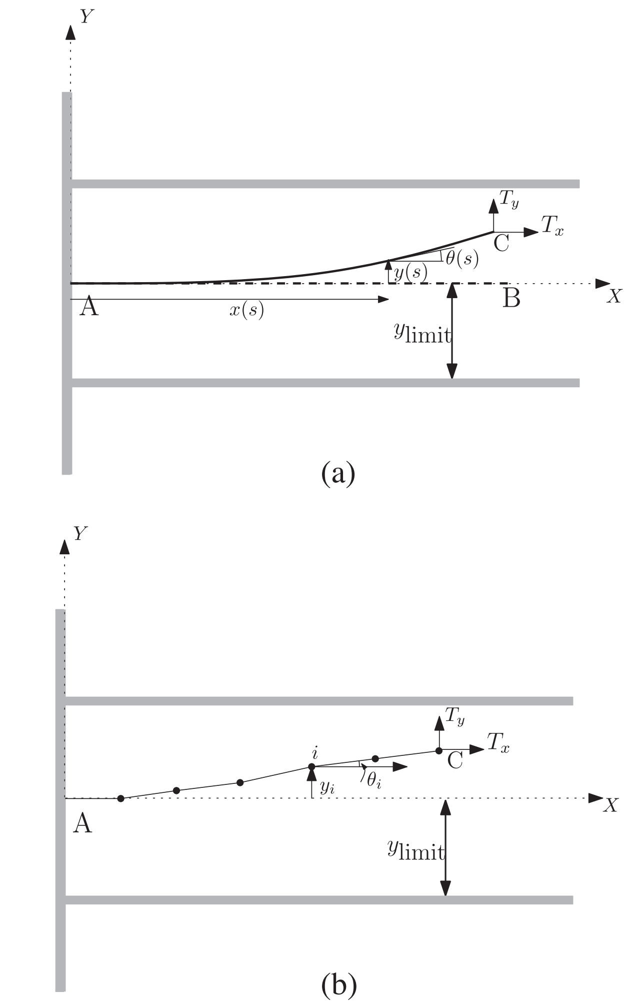

Fig. 1. The figure (a) shows a cantilever beam *AB* fixed at *A*, constrained to deform within two walls separated by a distance of $2y_{\text{limit}}$. Forces T_x and T_y act at the free end *B* as shown. Figure (b) shows the discretized version of the beam, with the dots representing the nodes of the discretized beam.

problem or as a starting point for using other convex optimization algorithms (see Section 1.4.1 of Boyd and Vandenberghe (2004)). Indeed, the latter path provides excellent dividends as can be seen in Section 6.

The particular problem that we are going to consider in this paper (namely that of a planar Euler beam fixed at one end and free at the other constrained between two parallel frictionless walls with a given separating distance) is not new (see Domokos et al., 1997; Holmes et al., 1999). In Holmes et al. (1999) a related problem of a planar simply supported beam is considered and a procedure for finding local equilibria is laid out, based on the notion of a "hybrid dynamical system". It has been well documented (see e.g. Flaherty and Keller, 1973), that based on the loading, solutions with different types of contact with the side walls appear. The central idea behind the approach developed in Holmes et al. (1999) is the classification of different kinds of behavior of the beam solutions with no contact (unconstrained), solutions with one point of contact, solutions with a whole line in contact, solutions with combination of points and lines of contact etc. They introduce the notion of solution classes or sheets to specify the particular type of branch that can arise.

The aim of this paper is not to carry out a complete investigation of this kind but to illustrate a different strategy that might possibly be used for a larger variety of problems. The approach proposed here is based on the following "direct minimization" method: "Given a large but finite number of possible configurations for a discretized version of the beam, find the ones with the lowest energy by direct search". This strategy will make sense only if (1) the number of configurations chosen is sufficiently large that it can approximate the continuum of possibilities to a reasonable degree of satisfaction and (2) an efficient means for searching through these configurations that does not suffer from exponential growth problems, can be found. The first problem can be dealt with by considering a simple "range discretization scheme" wherein the beam is divided into *N* nodes and each node takes on one of *M* possible *y* displacements. While the first step is routine in any numerical scheme, it is the second step that is unique to the method proposed here. By choosing *M* sufficiently large while at the same time restricting the possible values of *y* to be within the allowed values, we can simultaneously impose the unilateral wall constraints and approach any possible allowed configuration of the beam as closely as we wish. We then exploit the fact that the energy functional of the beam possesses a special "Markov structure" that enables rapid searching (using a dynamical programming technique) in polynomial time rather than exponential time.

In this paper, we will illustrate this technique be considering a discrete set of configurations that satisfy the wall constraints alone with no specific restriction being placed on the number or type of contact with the wall. The proposed approach then answers the question: "For a given load, the set of allowed configurations, which configuration(s) has the least energy". Admittedly this particular problem statement does not capture all the equilibrium states, since it ignores the local minima. However, it is possible to find all the local minima (among the set of allowed configurations) by considering a modified version of the algorithm (see Appendix for details of such an algorithm). We do not pursue this route here because this will make the illustration of the algorithm very complex.

A question may arise, as to what is the use of finding only global minima for such problems when there is a likelihood of local minima which can also serve as stable equilibria. We first note that:

1. We are finding the global minimum for the constrained problem, not for the unconstrained problem.[2]
2. Our first aim is to show the feasibility of the algorithm for solving these kinds of problems without resorting to experimentation/guesswork.
3. For micro and nano beams where thermal fluctuations become a major issue, from a statistical mechanical point of view, the probability of occurrence of a particular configuration is proportional to $e^{(-\beta E)}$ where $\beta = 1/KT$ where T is the temperature and E is the energy of a particular beam configuration. It is then easy to show that the most likely configuration of the beam is the one which corresponds to the global minimum.

1.1. Organization of the paper

In this paper, the beam problem is formally stated in Section 2. In Section 3, the discretization required to use the Viterbi algorithm for the energy minimization problem is set up and based on this discretization, the labeling for a state is defined. The details of the Viterbi algorithm are given as an appendix to this paper. Following the setup for the Viterbi algorithm, a brief discussion of the convergence of this technique is discussed in Section 4. In Section 5, the two algorithms, active-set method and simulated annealing, are explained and implemented for the beam problem considered. Section 6 lists the specifications used for the simulations and presents the discussion of results of the comparison of different methods for a beam buckling example.

[2] For the unconstrained problem, the minima can be found analytically.