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Asymptotic analysis of perforated plates and membranes. Part 1: Static problems for small holes

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ABSTRACT

Static problems for the elastic plates and rods periodically perforated by small holes of different shapes are solved using the asymptotic approach based on the combination of the asymptotic technique and the multi-scale homogenization method. Using the asymptotic homogenization method the original boundary-value problem is reduced to the combination of two types of problems. First one is a recurrent system of unit cell problems with the conditions of periodic continuation. And the second problem is a homogenized boundary-value problem for the entire domain, characterized by the constant effective coefficients obtained from the solution of the unit cell problems. The combination of the perturbation method and the technique of successive approximations is applied for the solution of the unit cell problems. Taking into the account small size of holes the method of perturbation of the shape of the boundary and the Schwarz alternating method are used. The problems of torsion of a rod with perforated cross-section; deflection of the perforated membrane loaded by a normal load; and bending of perforated plates with circular and square holes are considered consecutively. The error of the applied asymptotic techniques is estimated and the high accuracy of the obtained solutions is demonstrated.

1. Introduction

The perforated plates and membranes are widely used in the numerous technical applications, some examples are shown in the Fig. 1. Modern technological processes use perforated structures in the form of nanopore materials, e.g., porous silicon, zeolites, etc.

The mechanical analysis of perforated structures is a very important and a rather complicated task. Significant number of mathematical studies deals with the analysis of the boundary-value problems in perforated regions. They utilize a variety of approaches, in particular, the G-, H- and Γ- convergence (Cioranescu and Paulin, 1979; Dal Maso and Murat, 1997; Duvaut, 1977; Lions, 1980; Nazarov and Slutskii, 2006; Oleynik et al., 1986, 1992), two-scale convergence (Calvo-Jurado and Casado-Díaz, 2002; Casado-Díaz, 2000), unfolding method (Cioranescu et al., 2006, 2008). It can be shown that all these approaches are equivalent for the linear or quasi-linear problems.

As a rule, the mathematical studies are devoted to formulation and proof of theorems of existence and uniqueness of the solution, and to the formalism and convergence of the asymptotic process. The analysis of the unit cell problems in these studies is limited by a proof of their solvability. The further treatment and applications to the mechanical problems of practical interest remain beyond these mathematical studies. And it is commonly assumed that the further applications should be based on use of some numerical algorithms, as a rule, on the FEM.

Extensive survey of literature on a study of the perforated membranes, plates and shells from the viewpoint of mechanics of plates and shells is given in the monographs by Grigolyuk and Fil'shtinsky (1970) and Lewinski and Telega (2000).

The properties of the homogenized operators are studied by a number of authors. For example, Berlyand (1983a,b) has shown that if perforations are located in the points of a square lattice, then the homogenized plate is orthotropic, and if the unit cells have a form of regular hexagons, then the homogenized plate is isotropic. The properties of the coefficients of the homogenized operators are investigated by Bakhvalov and Panasenko (1989) and Kalamkarov (1992).

In a number of works the solution of the unit cell problem is obtained under the assumption of small concentration of perforations, and the effective coefficients are found in the form of expansions in powers of the corresponding small parameter (Berlyand, 1983a,b; Berlyand and Chudinovich, 1983). In some

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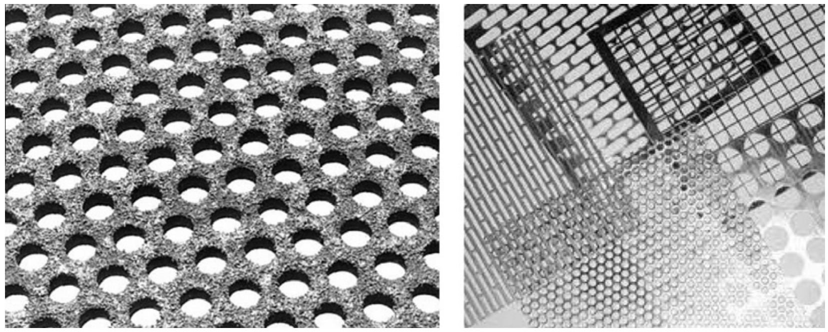

Fig. 1. Examples of perforated plates used in industrial applications.

particular cases the exact analytical solutions of the unit cell problems can be obtained, see Kalamkarov (1992). The explicit formulae for the effective stiffnesses are very useful, especially for the design and optimization of composite materials and structures, see Kalamkarov and Kolpakov (1997).

The use of technique of double-periodic analytical functions (Grigolyuk and Fil'shtinsky, 1970; Mokryakov, 2010) and integral equations (Helsing and Jonsson, 2003) makes it possible to reduce problems for perforated plates to the system of linear algebraic equations, regular or quasi-regular, which is solvable by the method of reduction.

The mean field theory is used by Shevlyakov and Skoblin (1993) to determine the effective bending characteristics of a plate, irregularly perforated by a large number of holes of a different shape. In particular, the effective characteristics of a plate, perforated by the elliptical holes are calculated. It is shown that the obtained approximate Young's and Shear moduli of the plate, perforated by the similar circular holes, are close to the exact values of these moduli determined in the case of a regularly perforated plate.

The problem of torsion of a perforated rod has been studied by Lukkassen et al. (2009). Authors used homogenized approach and they solved the unit cell problem numerically.

Movchan et al. (2002) used a lattice approximation in their study of dynamics of the perforated membranes.

The main purpose of the present work is to develop the mathematically justified methods of solution of the static and dynamic problems for the plates and membranes densely perforated by holes of different shapes. Our approach is based on the combination of the asymptotic technique and the multi-scale homogenization method. The basic idea of this approach is the following. In order to solve the original problem the domain of a plate is considered to be composed from a large number of characteristic periodic sections, the unit cells. And a small non-dimensional parameter is introduced as the ratio of the characteristic dimension of the unit cell to the smallest characteristic dimension of the entire domain. Further, using the multi-scale homogenization method, see, e.g., Bensoussan et al. (1978), Sanchez-Palencia (1980), Bakhvalov and Panasenko (1989), Duvaut (1977), Kalamkarov (1992) and Kalamkarov et al. (2009), the original boundary-value problem is reduced to the combination of two types of problems. First one is a recurrent system of unit cell problems with the conditions of periodic continuation. And the second problem is a homogenized boundary-value problem for the entire domain, characterized by the constant effective coefficients obtained from the solution of the unit cell problems. The combination of the methods of perturbation of the shape of boundary and the technique of successive approximations is applied for the solution of the unit cell problems. If the holes are small, then they are first examined in the infinite region, i.e., only the conditions on the internal boundary of the unit cell are taken into account. Solution of this problem is obtained using the method of perturbation of the shape of boundary (Guz and Nemish, 1987). According to this approach the internal boundary of the unit cell in the first approximation is substituted by a circle, and further the real shape of the internal boundary is taken into account in the subsequent approximations. The solutions obtain in this way leave some discrepancies in the conditions of the periodic continuation on the outer boundary of the unit cell. These discrepancies are compensated by the solution of the boundary-value problem for the domain without perforations, in which only the conditions on the outer boundary of the unit cell are taken into account. Note that the above described approach basically represents the realization of the Schwarz alternating method (Kantorovich and Krylov, 1958).

Present paper is dealing with the static problems for the perforated plates and membranes with small holes. Static and dynamic problems for the perforated plates and membranes with large holes are addresses in the Part 2 of the present work, see Andrianov et al. (2012).

Following this introduction, the rest of the paper is organized as follows: the asymptotic technique used in the present work is introduced in the Section 2 by an example of the problem of torsion of a rod with perforated cross-section. The obtained solution also describes the deflection of the perforated membrane loaded by a normal load. In the Section 3, the problem of bending of the perforated plates with circular and square holes is analyzed. The error estimation is discussed in the Section 4. Finally, conclusions and generalizations in the application of the asymptotic homogenization method are presented in the Section 5. The regularization of the solutions for the infinite domain is provided in the Appendix A.

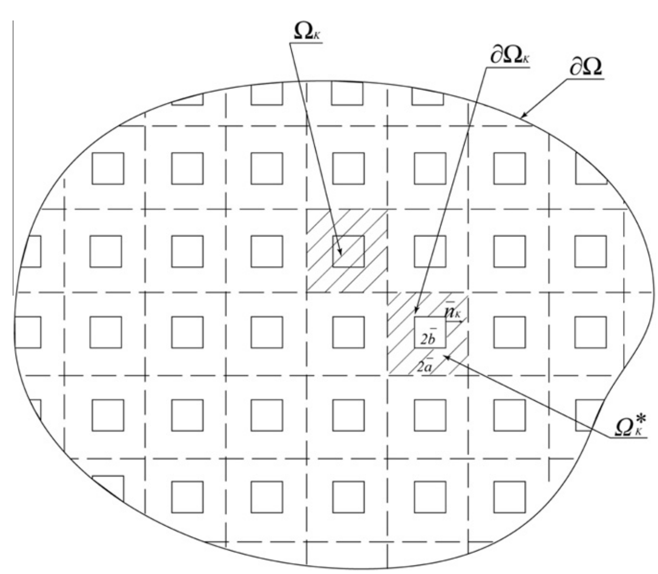

Fig. 2. Cross section of a rod periodically perforated by the square holes.