International Journal of Solids and Structures 49 (2012) 311–317

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Asymptotic analysis of perforated plates and membranes. Part 2: Static and dynamic problems for large holes

I.V. Andrianov [a], V.V. Danishevs'kyy [b], A.L. Kalamkarov [c,*]

[a] Institute of General Mechanics, RWTH Aachen, Templergraben 64, Aachen D-52062, Germany
[b] Prydniprovska State Academy of Civil Engineering and Architecture, 24-a Chernyshevskogo St., Dnipropetrovsk 49600, Ukraine
[c] Department of Mechanical Engineering, Dalhousie University Halifax, Nova Scotia, Canada B3J 2X4

ARTICLE INFO

Article history:
Available online 12 October 2011

Keywords:
Perforated plate and membrane
Asymptotic homogenization
Singular perturbation
Two-point Padé approximants

ABSTRACT

Static and dynamic problems for the elastic plates and membranes periodically perforated by holes of different shapes are solved using the combination of the singular perturbation technique and the multiscale asymptotic homogenization method. The problems of bending and vibration of perforated plates are considered. Using the asymptotic homogenization method the original boundary-value problems are reduced to the combination of two types of problems. First one is a recurrent system of unit cell problems with the conditions of periodic continuation. And the second problem is a homogenized boundary-value problem for the entire domain, characterized by the constant effective coefficients obtained from the solution of the unit cell problems. In the present paper the perforated plates with large holes are considered, and the singular perturbation method is used to solve the pertinent unit cell problems. Matching of limiting solutions for small and large holes using the two-point Padé approximants is also accomplished, and the analytical expressions for the effective stiffnesses of perforated plates with holes of arbitrary sizes are obtained.

1. Introduction

Extensive review of literature on a study of the perforated plates is given in the monographs by Grigolyuk and Fil'shtinskii (1970) and Lewinski and Telega (2000). In the present work we examine the periodically perforated plates and membranes, and we formulate and analyze the problem of determining the effective characteristics.

In the analysis of perforated plates and shells one should remember that their equations are obtained by the asymptotic simplification of the theory of elasticity equations under the assumption that the thickness of a plate h tends to zero. In the problem of periodically perforated plate, in addition to the small parameter h, there is another small parameter ε, which characterizes small size of holes and a small distance between them. In this setting the problem is analyzed by Lewinski and Telega (2000). In particular, it is shown that if $\varepsilon \gg h$ it is possible first to make a transfer from the theory of elasticity to the theory of plates, and then to apply the perturbation method to solve a problem of perforated plate. Furthermore, it is necessary that the minimum distance between the neighboring holes is substantially larger than h. The case $\varepsilon \sim h$ is analyzed in Kalamkarov (1992), where the general asymptotic homogenization composite shell model is developed; see also Kalamkarov et al. (2009) for a review on this model and its numerous applications.

The use of technique of double-periodic analytical functions, see, e.g., Grigolyuk and Fil'shtinskii (1970) and Mokryakov (2010), makes it possible to reduce problems for perforated plates to a regular or quasi-regular system of linear algebraic equations, which is solvable by the method of reduction.

The problem of determining the effective characteristics for Laplace's equation in the region perforated by the circular holes is examined in Balagurov (2001a). Complex potential outside of the holes or inclusions is expressed in terms of the Weierstrass zeta function and its derivatives. The infinite system of linear algebraic equations, which can be solved numerically, is obtained for the unknown coefficients. Different generalizations of this approach to the cases of holes and inclusions of arbitrary shapes are examined in Balagurov (2001b).

Boundary integral equations and the Fredholm integral equation of the second kind with non-singular operators are used in Helsing and Jonsson (2003) and Englund and Helsing (2001).

In the case of large holes the perforated plates are frequently substituted by the lattice (or framework) structures, see Brovko and Ilyushin (1993), and Nagayev and Hodjayev (1973). Cellular and network plates and shells of a regular structure are analyzed by Kalamkarov (1987, 1992) using the asymptotic homogenization composite shell model, see also Kalamkarov and Kolpakov (1997) and Kalamkarov et al. (2009).

* Corresponding author. Tel.: +1 902 494 6072; fax: +1 902 423 6711.
E-mail addresses: andrianov@iam.rwth-aachen.de (I.V. Andrianov), vdanish@ukr.net (V.V. Danishevs'kyy), alex.kalamkarov@dal.ca (A.L. Kalamkarov).

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doi:10.1016/j.ijsolstr.2011.10.004

The average field theory is applied by Pan'kov (1998) to determine the effective bending characteristics of a plate irregularly perforated by a large number of holes of the various shapes. In particular, the effective characteristics of a plate, perforated by the elliptical holes are calculated. It is shown that the obtained approximate Young's and Shear moduli of the plate perforated by the similar circular holes, have values, close to the exact values of these moduli determined in the case of a regularly perforated plate.

The influence of size of circular holes on the effective characteristics of the perforated rectangular plates was studied experimentally in Ivanov and Ivanov (1960) and Yakovlev (1954).

The problems of vibration of the perforated plates are important from the practical viewpoint, see, e.g., Preobrazhenskii (1980a,b) and Lim and Liew (1995).

The propagation of the out-of-plane time-harmonic waves in the isotropic elastic medium with an array of infinitely long circular cylindrical cavities is studied by Movchan et al. (2002) with the use of the generalized Rayleigh method. Authors also analyze a possibility of replacing the original perforated medium by a discrete model for a system of two types of particles connected by the springs.

The main purpose of the present work is to develop the mathematically justified methods of solution of the static and dynamic problems for the membranes and plates periodically perforated by holes of circle or square shapes. Our approach is based on the combination of the singular perturbation technique and the multi-scale asymptotic homogenization method. The basic idea of this approach is the following. In order to solve the original problem the domain of a plate is considered to be composed from a large number of the characteristic periodic sections, the unit cells. And a small non-dimensional parameter is introduced as the ratio of the characteristic dimension of the unit cell to the smallest characteristic dimension of the entire domain. Further, using the multi-scale homogenization method, see, e.g., Bensoussan et al. (1978), Duvaut (1977), Kalamkarov (1992) and Kalamkarov et al. (2009), the original boundary-value problem is reduced to the combination of two types of problems. First one is a recurrent system of the unit cell problems with the conditions of periodic continuation. And the second problem is a homogenized boundary-value problem for the entire domain, characterized by the constant effective coefficients obtained from the solution of the unit cell problems. Static problems in the case of small holes were studied in the Part 1 of the present work, see Andrianov et al. (in press). In the present paper we analyze the static and dynamic problems for the perforated plates and membranes in the opposite limiting case, namely, in the case of large holes. In this case for solving the unit cell problems we use the singular perturbation technique. Matching of limiting solutions for small and large holes using the two-point Padé approximants is also accomplished, and the analytical expressions for the effective stiffnesses of perforated plates with holes of arbitrary sizes are obtained.

Following this introduction, the rest of the paper is organized as follows: a problem of bending of a rectangular plate, periodically perforated by the square holes is considered in Section 2, where the asymptotic homogenization technique used in the present work is introduced. In the Section 3 we analyze the perforated plates with large holes. Matching of limiting solutions for small and large holes using the two-point Padé approximants is described in Section 4. Vibrations of perforated plates are studied in Section 5. Finally, the conclusions, generalizations and open problems are presented in the Section 6.

2. Bending of a rectangular plate, periodically perforated by the square holes

Considering a problem of bending of a rectangular plate, periodically perforated by the square holes, see Fig. 1. On this example we will introduce the asymptotic homogenization technique that will be applied in the present work.

Deflection of the plate satisfies the following biharmonic equation:

$$\Delta^2 w \equiv w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy} = q(x,y)/D, \tag{2.1}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $D = \frac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity, h is the thickness of plate, E is the Young's modulus, ν is the Poisson's ratio, and $q(x,y)$ is external transverse load.

Assume that the boundaries of the holes $\partial\Omega_k$ are free of loads, therefore

$$-(\Delta w)_{,\mathbf{n}_k} + (1-\nu)\left[\frac{1}{2}(w_{,xx} - w_{,yy})\sin 2\theta - w_{,xy}\cos 2\theta\right]_{\mathbf{s}_k} = 0 \quad \text{on } \partial\Omega_k, \tag{2.2}$$

$$M_n = \Delta w + (1-\nu)(w_{,xy}\sin 2\theta - w_{,xx}\sin^2\theta - w_{,yy}\cos^2\theta) = 0 \quad \text{on } \partial\Omega_k, \tag{2.3}$$

where θ is the angle between the x-axis and the normal $\mathbf{n}_k$ to the boundary of the hole; and $\mathbf{s}_k$ is a tangential vector to the boundary of the hole.

Without compromising generality assume that the outer boundary of the plate $\partial\Omega$ is simply supported, i.e.,

$$w = 0, \; M_n = 0 \quad \text{on } \partial\Omega. \tag{2.4}$$

Note that the boundary conditions of other types are also possible. If they don't restrict a movement of the plate as a solid body, the external loading must be self balanced.

Subdivide the plate into the periodic set of sections Ω_κ each of them containing a single hole, as it is shown in the Fig. 1. And assume that the characteristic period of external load $q(x,y)$ and the smallest characteristic dimension of plate L are substantially larger than the size of the periodicity cell, so that $\varepsilon = 2\bar{a}/L \ll 1$.

As a first step in asymptotic homogenization procedure, we introduce the "rapid" variables

$$\xi = \frac{x}{\varepsilon}; \quad \eta = \frac{y}{\varepsilon}. \tag{2.5}$$

For the "slow" variables we leave the above notation x and y.

The derivatives with respect to the coordinates o and y and the normal will be transformed as follows:

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial x} + \varepsilon^{-1}\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y} \to \frac{\partial}{\partial y} + \varepsilon^{-1}\frac{\partial}{\partial \eta}, \tag{2.6}$$

$$\frac{\partial}{\partial \mathbf{n}_k(x,y)} \to \mathbf{n}_k(\xi,\eta)\left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}\right) + \varepsilon^{-1}\mathbf{n}_k(\xi,\eta)\left(\mathbf{i}\frac{\partial}{\partial \xi} + \mathbf{j}\frac{\partial}{\partial \eta}\right), \tag{2.7}$$

where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of coordinates ξ and η.

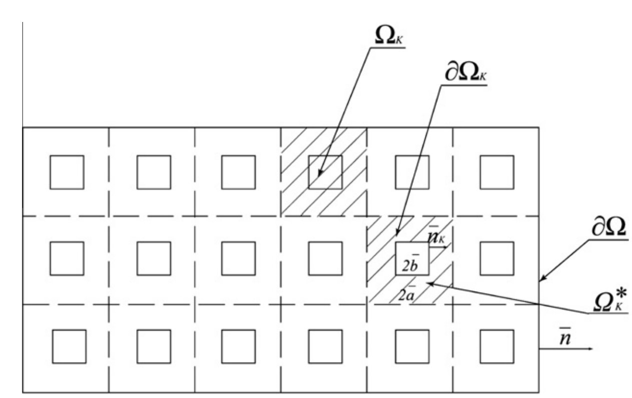

Fig. 1. Perforated plate with square holes.