International Journal of Solids and Structures 48 (2011) 1644–1657

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Shape features and finite element model updating from full-field strain data

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ARTICLE INFO

Article history:
Received 14 October 2010
Received in revised form 18 January 2011
Available online 15 February 2011

Keywords:
Full-field strain pattern
Shape features
Zernike moment descriptor
Model updating

ABSTRACT

Finite element model updating is an inverse problem based on measured structural outputs, in this case maximum principal strain measured using digital image correlation. Full-field responses in the form of strain maps contain valuable information for model updating but within large volumes of highly-redundant data. In this paper, shape descriptors based on Zernike polynomials having the properties of orthogonality and rotational invariance are shown to be powerful decomposition kernels for defining the shape or map of the strain distribution. A square plate with a circular hole subject to a uniaxial tensile load is considered and effective shape features are constructed using a set of modified Zernike polynomials. The modification includes the application of a decaying weighting function to the Zernike polynomials so that high strain magnitudes around the hole are well-represented. The Gram–Schmidt process is then used to ensure orthogonality for the obtained decomposition kernels over the domain of the specimen, i.e. excluding the hole. Results show that only a very small number of Zernike moment descriptors are necessary and sufficient to represent the full-field data. The onset of yielding may be quantified using the descriptors. Furthermore, model updating of nonlinear elasto-plastic material properties is carried out using the Zernike moment descriptors derived from full-field strain measurements.

1. Introduction

Traditionally validation of numerical models using data from experiments has relied on strain values obtained at a point or collection of isolated points using electrical strain gauges. Occasionally techniques such as photoelasticity, Moiré or holographic interferometry were used and data obtained along sections. Validation procedures were unsophisticated and largely consisted of qualitative assessment of the correlation between the data from experiments and the numerical model for 'hot spots' in the data where the stress in the model was observed to reach a maximum. In general, strain gauges were only placed at the 'hot-spots' indicated by the numerical model thus providing the possibility that other 'hot-spots' not found by the model could exist and be ignored. In addition, in lightweight structures, there is the possibility, that to save weight, material could be removed from a design in an area where the model indicates low or zero stress, has not been validated and is potentially incorrect. These circumstances could be characterised by an insufficiency of experimental data which fails to place sufficient demands upon the method of comparison of the experimental and simulated data. Recent advances in optical methods Sharpe (2008) permit full-field maps of surface strain to be obtained relatively easily using a variety of techniques, including digital image correlation, automated photoelasticity, electronic speckle pattern interferometry and thermoelastic stress analysis. These maps provide a level of redundancy in the data and require more sophisticated approaches to data comparison between experiments and numerical models Ravichandran et al. (2007), which is the focus of this work. The conceptual framework for verification and validation of computational models in solid mechanics is provided in a set of ASME guidelines (2006), and Schwer (2007). In this context verification refers to ascertaining that the computational model employed accurately reproduces the underlying mathematical model whereas validation refers to checking the extent to which the model is an accurate representation of the real world. From an experimentalist's perspective, the guidelines provide a sequence of steps starting from designing the experiment for the purpose of performing the validation through to quantifying the uncertainty in the data measured in the experiment. Earlier work Whelan et al. (2008), and Patterson et al. (2007) focused on calibration of the optical system of strain measurement in order to allow the measurement uncertainties to be quantified. Once data have been obtained from specifically-

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doi:10.1016/j.ijsolstr.2011.02.010

designed experiments and the uncertainty in the data quantified then the next step in the validation process is a detailed, quantitative comparison of these data with those obtained from the computational model which is to be validated. The experimental data might typically consist of two million values per strain component for each loading case considered, so a detailed and meaningful comparison is not straightforward.

The use of experimental measurements to ‘tune’ the parameters of a finite element (FE) model, generally known as model updating and applied in structural dynamics by Mottershead and Friswell (1993), and Friswell and Mottershead (1995), has been used recently to adjust material properties using full-field displacement data (and derived strain data) from digital image correlation (DIC) e.g. Avril et al. (2008). Typical examples include Lecompte et al. (2007), and Leclerc et al. (2009) who compare the full-field measurement and FE prediction directly. The FE nodes generally do not coincide with any of the available DIC measurement points and it is therefore necessary to project the DIC data onto the FE mesh. One approach is to fit a polynomial, usually the FE shape function, which may be implemented in DIC routines. Leclerc et al. (2009) showed that use of a coarse mesh had the effect of eliminating (or smoothing) the measurement noise, whereas a fine mesh, which was able to represent a complicated displacement field, was affected detrimentally by noise. Réthoré (2010) used a regularisation term for noise sensitivity minimisation to compute the bulk displacement field without boundary conditions. An alternative approach would be to parameterise the boundary conditions, so that they may be adjusted together with the constitutive equation terms by model updating (Ahmadian et al., 2002). A different approach is presented in the present paper, based on pattern recognition techniques (Jain and Duin, 2000) which offers a powerful approach with the following advantages: (1) the comparison of the FE prediction to the DIC measurement is made in terms of the distance between shape feature vectors and therefore does not require projection of the measurement onto the FE mesh – the DIC evaluation grid does not have to be the same as the FE mesh; (2) the data is reduced to a small number of most significant shape features so that inherent redundancy in the full-field measurement is eliminated; and (3) the image is decomposed using an orthogonal basis with wavelengths that are much longer than the characteristic dimension of the noise, which (assuming random noise with zero mean) is therefore rejected.

A general technique in pattern recognition is to extract useful features from the pattern being analysed by different means and to make the comparison based on features (Duda et al., 1973). Functional transformation is one of the most adopted procedures to extract such features. Using the Fourier transform to extract shape features is one the earliest techniques in pattern recognition by Zahn and Roskies (1972), and Persoon and Fu (1977). Global features are captured by the lower spatial frequency components while the noise is present in the higher frequency components. Wavelet descriptors are especially useful in distinguishing strain-field heterogeneities (Mallat and Hwang, 1992). A number of papers address the problem of structural damage detection using the wavelet descriptor, including Chang (2004), Fan and Qiao (2009), Douka (2003), and Liew and Wang (1998). Multi-resolution analysis proposed by Mallat (1989) is an important capability of wavelets, having great potential for data compression. Digital image coding using wavelets was described by Antonini et al. (1992).

The geometric moments of binary images, introduced by Hu (1962), were probably the first image moment descriptors. Orthogonal image moment descriptors were later proposed by Teague (1980) using the continuous orthogonal polynomials – e.g. the Zernike and Legendre polynomials. Discrete orthogonal polynomials were proposed to reduce the numerical errors produced by the discretisation of the continuous polynomials during the moment evaluation of the digital images. Mukundan et al. (2001) adopted the discrete Tchebichef polynomials to characterise the global shape features of the digital images. Yap et al. (2003) introduce the discrete Krawtchouk moment which is capable in describing local features. The discrete Hahn moment descriptor (Yap et al., 2007) is more general than the Tchebichef moments (global characteristics) and the Krawtchouk moments (local characteristics) and has characteristics between them These orthogonal polynomials are defined on uniformly distributed lattices. The use of discrete orthogonal polynomials defined on non-uniform lattices for image analysis was proposed by Zhu and his co-workers (Zhu et al., 2007b,a). The dual Hahn and the Racah polynomials have promising possibilities for those applications when the DIC evaluations or FE predictions are performed on non-uniform meshes.

Image processing methods have been investigated and applied to recognise full-field vibration mode shapes Wang et al. (2009b,a, 2011). It was found that suitable selection of transformation kernel functions produced succinct and efficient features, making the comparison of redundant full-field vibration mode shapes with predicted eigenvectors achievable in a concise and effective way. It is intended to apply such techniques to the full-field stress/strain pattern in this paper.

The selection of the kernel functions is always problem dependent. However, certain general requirements for the shape feature extraction kernels should be satisfied: unique definition, geometric invariance (scaling, rotation, reflection etc.), ease of reconstruction, inexpensive computation and so on. The Zernike polynomial, one of the most powerful feature extraction kernels for circular and annular structures (Wang et al., 2009b), fulfils several requirements as mentioned – rotational invariance, uniqueness and ease of retrieval by using orthogonality conditions. The features extracted by the Zernike polynomials are called the Zernike Moments Descriptor (ZMD). However, the orthogonality of the Zernike polynomials is satisfied only within the domain of the unit circle. It is necessary to be able to create or modify a set of orthogonal kernel functions defined on non-regular domains, in the present case a rectangular plate with a circular hole. The Gram–Schmidt orthogonalisation (GSO) process was proposed Mahajan (1981) to regain the orthogonality of the Zernike polynomials defined over an annular domain. This approach is readily extended to non-circular structures.

The modified Zernike polynomials developed in this paper are ideally suited to the extraction of shape features from cyclically symmetric strain patterns, such as in the particular study of an aluminium plate with a circular hole under tensile loading. Modification of the Zernike kernels is optimised by Newton’s method. The resulting ZMs show the required efficiency and effectiveness as discussed in Section 3. The full-field strain contours are seen to ‘expand’ with increasing load and it is seen that this can be represented by the development of a very small number of the ZMDs. Furthermore, finite element model updating of the elasto-plastic material properties for the specimen is also successfully carried out based on the experimental data and using the most significant ZMDs, as explained in Section 4.

2. Construction of shape features – modified Zernike moment descriptors

The general form of transform-based shape features may be expressed as,

$$\mathcal{D} = \mathcal{T}[\mathcal{S}(x,y)] \tag{1}$$

where $\mathcal{S}(x,y)$ denotes the continuous shape pattern and $\mathcal{T}[*]$ represents the transformation for extracting the shape features. More specifically, it can be defined by projecting the pattern onto a set of kernel functions $\mathcal{R}_i(x,y)_{i=1,2,\ldots}$ as