International Journal of Solids and Structures 48 (2011) 1738–1753

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Finite strain hyperelastoplastic modelling of saturated porous media with compressible constituents

A. Gajo *

Dipartimento di Ingegneria Meccanica e Strutturale, Università di Trento, Via Mesiano 77, 38050 Povo, Trento, Italy

ARTICLE INFO

Article history:
Received 16 November 2010
Received in revised form 23 February 2011
Available online 3 March 2011

Keywords:
Porous media
Finite strains
Elastoplasticity
Compressible constituents
Hyperelasticity

ABSTRACT

Starting from the general expression of the free-energy function of a saturated porous medium at finite deformations in the case of compressible fluid and solid constituents, and from the internal dissipation increment, the general expressions of the plastic potential and flow rule are deduced together with the general form of the consistency condition. Reference is made to an elementary volume moving with the solid skeleton in a Lagrangian description, which is treated as an open system from which the pore fluid can flow freely in and out.

As a result, a *generalisation* is provided of the classical Prandtl–Reuss relationship of small strain elastoplasticity in single-phase media to finite strain *multiplicative* (for $\boldsymbol{F}$) and *additive* (for the fluid mass content) elastoplasticity in saturated porous media with compressible constituents.

The following particular cases are analysed in detail: null plastic volume change of the solid constituent, incompressibility of the solid constituent, incompressibility of both fluid and solid constituents, quasi-linear theory (in which the solid constituent is assumed to be nearly incompressible, and therefore undergoing small volume changes), and geometrically linearised theory. The simplified approaches previously presented in the literature are thus recovered within a unified framework and new, simplified constitutive assumptions are made.

1. Introduction

Papers dealing with the elastoplastic behaviour of porous media at finite strains typically reflect the two historical approaches in the literature to the constitutive modelling of porous media, namely: mixture theories and the so-called ‘purely macroscale theories’. In the former, the porous media are represented by superposed and interacting continua and the field equations of each constituent are derived from averaging processes (Morland, 1972; Bowen, 1976, 1982), whereas the latter were first proposed by Biot (1972, 1973, 1977) and later by Coussy (1989), and assume that the standard concepts of continuum mechanics are still relevant on a macroscale (Coussy et al., 1998). It is worth recalling that it is generally accepted that an equation is missing in mixture theory for a saturated porous medium (e.g. Svendsen and Hutter, 1995), so several approaches have been suggested to overcome this drawback (see De Boer, 1996, for a review of the various proposals). Within this framework, Gray and Miller (2005) have recently proposed a thermodynamically constrained averaging theory starting from the microscale constituent continua to the macroscale. This approach was later applied by Gray and Schrefler (2007) to small-strain, multiphase media, recovering the traditional form of Biot’s coefficients.

Within the framework of mixture theories, the first papers taking the elastoplastic behaviour of the solid material into account are those by Morland (1972), Kojić and Cheatham (1974), De Boer and Kowalski (1983), and De Boer and Ehlers (1986), each of them based on different simplifying assumptions, such as geometrically linearised theory, the special multiplicative decomposition of the finite deformation gradient $\boldsymbol{F}$, the incompressibility of the fluid and solid constituents, or the small, elastic strains of the solid constituent. Among the first works based on ‘purely macroscale theories’, there are those by Carter et al. (1977, 1979) and Prevost (1980), that consider incompressible solid constituents and are based on the Jaumann stress rate in an updated Lagrangian approach (see also Meroi et al., 1995). In particular, the latter assumption may lead to unphysical responses (Johnson and Bammann, 1984), and that is why the most recent solutions are based on hyperelastic formulations.

It is generally agreed that the assumption of incompressibility of the solid grains *immensely* simplifies the relationships (Bennethum, 2006), because the *coupling* between the solid and fluid phases is much weaker in this case. That is why the compressibility of the solid constituent has mostly been neglected in the

* Tel.: +39 0461 882519; fax: +39 0461 882599.
E-mail address: alessandro.gajo@ing.unitn.it
URL: http://www.ing.unitn.it/~gajoal/

0020-7683/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2011.02.021

engineering applications and constitutive models for considering porous media at finite strain proposed so far. For instance, Ehlers (1991), Borja and Alarcon (1995), Diebels and Ehlers (1996) and Larsson and Larsson (2002), Sanavia et al. (2002) all considered incompressible solid constituents. Alternatively, the compressibility of the solid constituents has been considered by means of simplifying assumptions: Advani et al. (1993) and De Boer and Bluhm (1999) adopted the semilinear theory suggested by Biot (1973), Coussy (1995) proposed a *linear* extension of Biot's classical poroelasticity to *small* (not necessarily infinitesimal) deformations, Armero (1999) treated the compressibility of the constituents in a simplified way, considering Biot's parameter as a constant (as in linear theory), Bernaud et al. (2002) considered small elastic strains associated with large plastic deformations, thus obtaining constant Biot coefficients, Larsson et al. (2004) identified the solid constituent with the fibre bundles being wetted with resin in the processing of fibre composite materials and neglected the role of the intergranular stress on the compaction of the solid constituent, whereas Borja (2006) proposed a geometrically nonlinear theoretical framework based on the current configuration in which the definition of effective stress proposed by Nur and Byerlee (1971) for small strains is recovered with non-constant coefficients.

The compressibility of the solid constituents within an elastoplastic model of porous media at finite strains was considered by Ehlers (1993), who provided the general functional dependence of constitutive relationships within the framework of mixture theories for second-grade porous materials, working on the simplifying assumption that the solid constituents can only undergo reversible volume changes. Unfortunately, the effects of the microscopic volume changes of the solid constituent on the macroscopic deformation of the solid skeleton were not discussed clearly.

Gajo (2010) recently proposed a fairly general method for defining the free energy density function of saturated porous materials with compressible solid and fluid constituents within the framework of 'purely macroscale theories'. Reference was made to an elementary volume moving with the solid skeleton in a Lagrangian description, so – in contrast with mixture theories – the free energy density is not the simple sum of the free energies of the single constituents. The aim of the present work is to exploit the general expression of the free energy density function proposed by Gajo (2010) to extend a widely-adopted, finite-strains, elastoplastic constitutive approach for single-phase media (e.g. Lubliner, 1990; Maugin, 1992; Simo, 1998) to saturated porous media; this is in the conviction that the proposed extension is not limited to the particular constitutive approach to single-phase media taken into account, although the definition of the 'plastic deformation rate' is somewhat restricted, because it is constrained by the need to provide consistent evaluations of the dissipation rate in the case of incompressible fluid constituent.

The elementary volume is treated as an open system from which the pore fluid can flow freely in and out. As a result, the strain-like variables are a suitable measure of the solid skeleton deformation and the pore fluid mass content, whereas the work conjugated stress-like quantities are the appropriate measure of the total stress (which is work-conjugated to the selected measure of the solid skeleton deformation) and the chemical potential of the pore fluid. The usual multiplicative decomposition of the deformation gradient $\boldsymbol{F}$ into an elastic $\boldsymbol{F}_e$ and a plastic $\boldsymbol{F}_p$ part is associated with the additive decomposition of the mass content of pore fluid into an elastic and a plastic part, as first proposed by Armero (1999). The resulting constitutive model takes large-strain microscopic, either reversible or irreversible, volume changes of the solid constituent into account. The classical Prandtl–Reuss relationships of the geometric linear theory for single-phase media are thus *generalised* to the finite-strain multiplicative elastoplasticity for saturated porous media with compressible constituents.

The content of the paper can be outlined as follows. The assumed kinematic relations with the usual intermediate configuration and the Clausius–Planck inequality are described in Sections 2 and 3, respectively, whereas the hyperelastic formulation proposed by Gajo (2010) is briefly recalled and extended to the context of irreversible deformations in Section 4. The Clausius–Planck inequality is subsequently exploited in Section 5 for deducing the basic structure of the elastoplastic relationships. The consistency condition and the hyperelastoplastic rates are given in Section 6, in the general case of irreversible volume changes of the solid constituents. In Appendix C the following particular cases are analysed in detail: (i) null plastic volume changes in the solid constituent; (ii) the incompressibility of the solid constituent; (iii) the incompressibility of both the fluid and the solid constituents; (iv) quasi-linear theory (in which the solid constituent is assumed to be nearly incompressible, thus undergoing small volume changes, Biot, 1973); and (v) geometrically linearised theory. As a particular case of item (v), the approach proposed by Loret and Harireche (1991) and by Coussy (2007) for saturated porous media at small strains with solid constituents undergoing only reversible volume changes is fully recovered. Although the constitutive relationships are proposed for the intermediate configuration, useful hints are given for defining the constitutive relationships in the reference or in the current configuration as well.

In the most general case, the constitutive equations turn out to have a coupling of elastic and plastic properties (i.e. elastoplastic coupling) associated with a non-symmetric tangent operator. The relationships are largely simplified for a null plastic volume change of the solid constituents (case i), however, or for the incompressibility of the solid constituents (cases ii and iii), because the tangent constitutive operator becomes symmetric due to the associated flow rule, despite the elastoplastic coupling, which disappears only in the case of geometrically linearised theory (case v), where the tangent constitutive operator is symmetric for any kind of behaviour of the solid constituent (provided the flow rule is associated).

Notation: four tensorial products will be used, that can be defined as follows:

$$(\boldsymbol{A}\otimes\boldsymbol{B})[\boldsymbol{C}]=(\boldsymbol{B}\cdot\boldsymbol{C})\boldsymbol{A}\quad\text{and}\quad(\boldsymbol{A}\,\overline{\underline{\otimes}}\,\boldsymbol{B})[\boldsymbol{C}]=\frac{1}{2}(\boldsymbol{ACB}^T+\boldsymbol{AC}^T\boldsymbol{B}^T),$$

$$(\boldsymbol{A}\,\overline{\otimes}\,\boldsymbol{B})[\boldsymbol{C}]=\boldsymbol{ACB}^T\quad\text{and}\quad(\boldsymbol{A}\,\underline{\otimes}\,\boldsymbol{B})[\boldsymbol{C}]=\boldsymbol{AC}^T\boldsymbol{B}^T,$$

for every second-order tensor $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$.

2. Kinematic relations

Let $\boldsymbol{x}=\boldsymbol{\varphi}(\boldsymbol{X},t)$ denote the current position (with $\boldsymbol{x}\in\mathcal{B}$ in the current configuration $\mathcal{B}$) of the solid skeleton particle having the initial position $\boldsymbol{X}\in\mathcal{B}_0$ in the reference configuration, $\mathcal{B}_0$. Moreover, let $\boldsymbol{F}=\mathrm{Grad}\,\boldsymbol{\varphi}$, and $J=\det\boldsymbol{F}$ denote the deformation gradient and its Jacobian, respectively, whereas $\boldsymbol{L}=\dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$ is the spatial gradient of the spatial velocity field (i.e. grad$\boldsymbol{v}$). The current infinitesimal volume $d\Omega$ of the solid skeleton is obviously related to its initial material volume $d\Omega_0$, through $d\Omega=Jd\Omega_0$.

The volume occupied by the pore fluid in the reference configuration is $n_0d\Omega_0$, where n_0 is the initial porosity in the reference configuration; let n denote the current porosity. If ρ_w and ρ_{w0} denote the current and the initial density of the pore fluid, respectively, let J_w denote their ratio, namely $J_w=\rho_{w0}/\rho_w$. The initial pore fluid mass content in the infinitesimal volume of solid skeleton $d\Omega_0$ is

$$d\tilde{m}_{w0}=\rho_{w0}n_0\,d\Omega_0, \tag{1}$$

whereas the fluid mass content in the current configuration is