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Oscillations for a delayed predator–prey model with Hassell–Varley-type functional response☆

Changjin Xu [a,*], Peiluan Li [b]

[a] Guizhou Key Laboratory of Economics System Simulation, School of Mathematics and Statistics, Guizhou College of Finance and Economics, Guiyang 550004, China
[b] School of Mathematics and Statistics, Henan University of Science and Technology, Luoyang, Henan 471023, China

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ABSTRACT

In this paper, a delayed predator–prey model with Hassell–Varley-type functional response is investigated. By choosing the delay as a bifurcation parameter and analyzing the locations on the complex plane of the roots of the associated characteristic equation, the existence of a bifurcation parameter point is determined. It is found that a Hopf bifurcation occurs when the parameter τ passes through a series of critical values. The direction and the stability of Hopf bifurcation periodic solutions are determined by using the normal form theory and the center manifold theorem due to Faria and Maglhalaes (1995). In addition, using a global Hopf bifurcation result of Wu (1998) for functional differential equations, we show the global existence of periodic solutions. Some numerical simulations are presented to substantiate the analytical results. Finally, some biological explanations and the main conclusions are included.

1. Introduction

Since the pioneering work of Volterra [1] and Lotka [2] in the mid-1920s, there has been increasing interest in investigating the dynamical behaviors of predator–prey models in both ecology and mathematical ecology [3–11]. In particular, one of the important dynamical predator–prey behaviors, such as periodic phenomena and bifurcation has become even more interesting [6–10,12–24]. In 1980, Freeman [25] proposed a most popular predator–prey model with Michaelis–Menten-type functional response:

$$\begin{cases} \dfrac{dx_1}{dt} = rx_1\left(1-\dfrac{x_1}{K}\right)-\dfrac{cx_1x_2}{m+x_1}, \\ \dfrac{dx_2}{dt} = x_2\left(-d+\dfrac{fx_1}{m+x_1}\right), \\ x(0)>0, y(0)>0, \end{cases} \tag{1}$$

where x_1, x_2 denote the population of preys and predators at time t, respectively. r, K, c, m, d, and f are positive constants that denote the prey's intrinsic growth rate, carrying capacity, capturing rate, half-saturation constant, predator death rate, maximal predator growth rate, respectively. For more details about the model, the reader is referred to [25].

Considering that in many situations, predators must search and share or compete for food, Arditi and Ginzburg

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* Corresponding author.
E-mail address: xcj403@126.com (C. Xu).

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[3] introduced and studied the following ratio-dependent-type functional response model:

$$\begin{cases} \dfrac{dx_1}{dt} = rx_1\left(1-\dfrac{x_1}{K}\right) - \dfrac{cx_1x_2}{mx_2+x_1}, \\ \dfrac{dx_2}{dt} = x_2\left(-d+\dfrac{fx_1}{mx_2+x_1}\right), \\ \qquad x(0)>0, y(0)>0 \end{cases} \tag{1.2}$$

Since the functional response depends on the predator density in a different way, Hassel and Varley [26] reconstructed the predator–prey model with Hassell–Varly-type functional response, which takes the following form:

$$\begin{cases} \dfrac{dx_1}{dt} = rx_1\left[1-\dfrac{x_1}{K}\right] - \dfrac{cx_1x_2}{mx_2^\gamma+x_1}, \\ \dfrac{dx_2}{dt} = x_2\left[-d+\dfrac{fx_1}{mx_2^\gamma+x_1}\right], \\ \qquad x(0)>0, y(0)>0, \end{cases} \tag{1.3}$$

where $\gamma \in (0,1)$ is called the HV constant. Generally, the consumptions of prey by the predator throughout its past history governs the present birth rate of the predator. Motivated by this point of view, Wang [27] introduced and investigated the periodic solutions to the following delayed predator–prey model:

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)\left[a(t)-b(t)x_1(t-\tau(t)) - \dfrac{c(t)x_2(t)}{mx_2^\gamma(t)+x_1(t)}\right], \\ \dfrac{dx_2(t)}{dt} = x_2(t)\left[-d(t)+\dfrac{r(t)x_1(t)}{mx_2^\gamma+x_1}\right], \\ \qquad x(0)0, y(0)>0 \end{cases} \tag{1.4}$$

with the following initial condition:

$$\begin{cases} x_1(t) = \varphi(\theta), \theta\in[-\delta,0], \varphi(0)=\varphi_0>0, \\ x_2(t) = \psi(\theta), \theta\in[-\delta,0], \psi(0)=\psi_0>0, \end{cases} \tag{1.5}$$

where $\delta = \sup_{t\in[0,\omega]}\{\tau(t)\}, \varphi, \psi \in C([-\delta,0])$ with the norm $\|x\| = \sup_{t\in[-\delta,0]}|x(t)|$. It is worth pointing out that during the course of the predator–prey interaction when predators do not form groups, one can assume that the HV constant is equal to 1, that is, $\gamma = 1$. Moreover, it is more reasonable to incorporate the delay into Hassell–Varly-type functional response. From the point of view of biology, we will consider the following model with delayed Hassell–Varly-type functional response:

$$\begin{cases} \dfrac{dx_1}{dt} = x_1\left[a-bx_1(t-\tau) - \dfrac{cx_2(t-\tau)}{mx_2(t-\tau)+x_1(t-\tau)}\right], \\ \dfrac{dx_2}{dt} = x_2\left[-d+\dfrac{rx_1(t-\tau)}{mx_2(t-\tau)+x_1(t-\tau)}\right] \end{cases} \tag{1.6}$$

In this paper, we will devote our attention to investigating the properties of a Hopf bifurcation of system (1.6), that is to say, we shall take the delay τ as the bifurcation parameter and show that when τ passes through a certain critical value, the positive equilibrium loses its stability and a Hopf bifurcation will take place. Furthermore, when the delay τ takes a sequence of critical values containing the above critical value, the positive equilibrium of system (1.6) will undergo a Hopf bifurcation. In particular, by using the normal form theory and the center manifold reduction due to Faria and Maglhalaes [28], the formulae for determining the direction of Hopf bifurcations and the stability of bifurcating periodic solutions are obtained. In addition, the existence of periodic solutions for τ far away from the Hopf bifurcation values is also established by means of the global Hopf bifurcation result of Wu [29].

In order to obtain the main results of our paper, throughout this paper, we assume that the coefficients of system (1.6) satisfy the following condition:

H_1. $am^2 + cd - cr > 0, r > d$

This paper is organized as follows. In Section 2, the stability of the positive equilibrium and the existence of a Hopf bifurcation at the positive equilibrium are studied. In Section 3, the direction of Hopf bifurcation and the stability of bifurcating periodic solutions on the center manifold are determined. In Section 4, numerical simulations are carried out to illustrate the validity of the main results. In Section 5, some conditions that guarantee the global existence of the bifurcating periodic solutions to the model are given. Biological explanations and some main conclusions are drawn in Section 6.

2. Stability of the equilibrium and existence of the local Hopf bifurcation

In the section, by analyzing the characteristic equation of the linearized system of system (1.6) at the positive equilibrium, we investigate the stability of the positive equilibrium and the existence of the local Hopf bifurcations occurring at the positive equilibrium.

Considering the biological meaning, we only study the property of a unique positive equilibrium (i.e., coexistence equilibrium). It is easy to see that under the hypothesis (H_1), system (1.6) has a unique positive equilibrium $E_*(x_1^*, x_2^*)$, where

$$x_1^* = \frac{am^2+cd-cr}{abm}, x_2^* = \frac{(r-d)(am^2+cd-cr)}{abdm^2}$$

Let $u_1(t) = x_1(t) - x_1^*, u_2(t) = x_2(t) - x_2^*$, then, system (1.6) takes the following form: