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Folding of fiber composites with a hyperelastic matrix

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ABSTRACT

This paper presents an experimental and numerical study of the folding behavior of thin composite materials consisting of carbon fibers embedded in a silicone matrix. The soft matrix allows the fibers to microbuckle without breaking and this acts as a stress relief mechanism during folding, which allows the material to reach very high curvatures. The experiments show a highly non-linear moment vs. curvature relationship, as well as strain softening under cyclic loading. A finite element model has been created to study the micromechanics of the problem. The fibers are modeled as linear-elastic solid elements distributed in a hyperelastic matrix according to a random arrangement based on experimental observations. The simulations obtained from this model capture the detailed micromechanics of the problem and the experimentally observed non-linear response. The proposed model is in good quantitative agreement with the experimental results for the case of lower fiber volume fractions but in the case of higher volume fractions the predicted response is overly stiff.

1. Introduction

Deployable space structures have traditionally been based around stiff structural members connected by mechanical joints, but more recently joint-less architectures have been proposed. In this case large relative motions between different parts of a structure are achieved by allowing the connecting elements to deform elastically. For example, deployable solar arrays consisting of sandwich panels connected by tape spring hinges make use of cylindrical thin shells to provide the connection between the panels. This connection becomes highly compliant when elastic folds are created in the tape springs, allowing the panels to stow compactly, but it becomes much stiffer when the tape springs are allowed to deploy and snap back into the original configuration. The maximum curvature of the tape springs cannot exceed a limiting value that is related to the failure strain of the material (Rimrott, 1966; Yee et al., 2004).

Tape spring hinges have been made from metal (Watt and Pellegrino, 2002; Auternaud et al., 1993), carbon fiber composites (Yee et al., 2004; Yee and Pellegrino, 2005), and memory matrix composites (Campbell et al., 2005; Barrett et al., 2006). This last approach makes use of polymers that become softer above a transition temperature and these materials have been exploited to make hinges that can be heated and folded to very high curvatures. When they are cooled the hinges remain "frozen" in the folded configuration but will then self-deploy when they are re-heated. A variant of this approach is to use a very soft elastic matrix, orders of magnitudes softer than standard epoxies; in this way the same type of behavior can be achieved without the need to heat the material. The reason is that the fibers on the compression side of the material can form elastic microbuckles and so, instead of being subject to a large compressive strain, they are mainly under bending.

This approach has already been exploited by Datashvili et al. (2005) who used a silicone matrix and by Mejia-Ariza et al. (2006) and Rehnmark et al. (2007) who used other approaches. Models showing exceptional folding capabilities have been demonstrated (Datashvili et al., 2010; Mejia-Ariza et al., 2010).

A number of simplified analytical models of this behavior have been already developed, see Section 2 of this paper, however these models do not capture the detailed micromechanics of the fibers and matrix and so their predictive capability is limited. Our long-term objective is to establish experimentally validated models for the folding behavior of a thin composite consisting of unidirectional carbon fibers embedded in a silicone matrix. Such models would be used to predict the strain in the fibers for any given, overall curvature and hence to estimate the failure curvature of any specimen with known properties and to study a variety of related effects, such as estimating the bending stiffness of a composite of stiff fibers embedded in a soft matrix, allowing for the softening associated with microbuckling.

A key question that arises when developing such models is what assumptions are justified to reach an effective compromise between model complexity and ability to obtain physically representative predictions. In the present study we focus on two key effects, fiber distribution and large strain hyperelastic behavior of the matrix without any damage or other dissipation sources.

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Damage effects have been considered in a separate paper (López Jiménez and Pellegrino, 2011).

The paper is laid out as follows. Section 2 presents a review of some relevant literature. Section 3 details the material properties and fabrication process, as well as an analysis of fiber distribution based on micrographs of actual specimens. Section 4 presents the experimental results of a folding test that shows evidence of microbuckling in the material; a characterization of the non-linear moment–curvature relationship of the material is also obtained. It is shown that, in addition to fiber microbuckling, there is evidence of damage-related stress-softening. Section 5 describes the finite element model used to study in detail the behavior of a periodic unit cell based on a representative volume element (RVE) including a small number of elastic fibers, typically between ten and thirty, embedded in a hyperelastic matrix. The RVE has a length longer than the buckle wavelength. Section 5.2 discusses two different fiber arrangements used in the simulations. The first one is a hexagonal pattern, characterized only by the fiber volume fraction in the material. The second starts from a random distribution of fibers that is rearranged until it matches the distribution observed in actual micrographs. The predictions for buckle wavelength and moment–curvature relationship obtained from the finite element model are compared with the experimental results in Section 6. Section 7 summarizes the findings of this work and concludes the paper.

2. Background

The literature relevant to the present study can be categorized under three main headings: elastic fiber microbuckling, large straining of soft materials with embedded fibers, and strain softening.

Regarding the first topic, fiber microbuckling is a well known failure mode for composite materials, see the review in Fleck (1997). Rosen (1965) proposed a model based on the buckling of a beam on an elastic foundation; he assumed a sinusoidal shape for the buckled fibers and compared the work done by the external forces with the strain energy in the system. He considered two different buckling modes. In the first one the matrix deforms in extension, while in the second one it shears. The second mode requires a lower critical load and it is therefore considered the preferred one. Rosen's approach of treating the fibers as beams on an elastic foundation has been used by several other authors to explore the effects of material thickness, fiber volume fraction and type of load, see for example Drapier et al. (1996, 1999, 2001) and Parnes and Chiskis (2002).

Microbuckling has been invoked as a stress relieving mechanism in thermoplastic and elastic memory composites that are subject to large bending curvatures (Campbell et al., 2004; Marissen and Brouwer, 1999; Murphey et al., 2001). Fig. 1 shows a sketch that explains this behavior. When folding starts, the

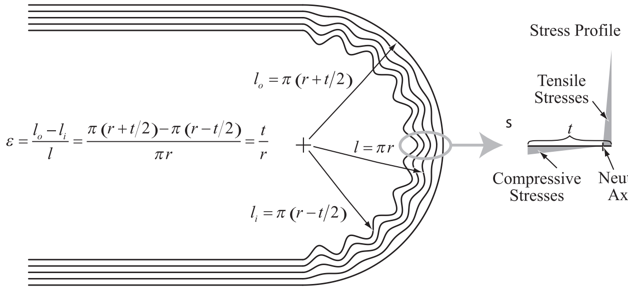

Fig. 1. Fiber microbuckling and stress profile in a heavily bent laminate, taken from Murphey et al. (2001).

extensional stiffness is uniform through the thickness and hence the neutral axis lies in the middle plane. As the fibers on the compression side reach the critical buckling load, their stiffness is reduced in the post-buckling range and this corresponds to a bilinear constitutive model that shifts the neutral axis towards the tensile side of the laminate. This effect reduces both the maximum tensile and the maximum compressive strain in the fibers, allowing them to remain elastic for much higher curvatures than if the fibers had remained unbuckled. It should be noted that in Fig. 1 the buckling deflection has been depicted as out of plane for a better visualization, but experimental observations show that in most cases it occurs within the plane of the material, that is, parallel to the axis of bending.

This effect has been studied by Francis et al. (2006) and Francis (2008) who provided the following approximation for the fiber buckle wavelength:

$$\lambda = \frac{\pi}{2}\left(\frac{9V_f t^2 d^2 E_f}{8G_m \ln\left(\frac{6t}{d}\sqrt{\frac{V_f}{\pi}}\right)}\right)^{\frac{1}{4}} \tag{1}$$

where d is the fiber diameter, t is the thickness of composite plate being bent, E_f is the stiffness of the fibers, G_m is the shear modulus of the matrix, and V_f is the fiber volume fraction.

The amplitude of the buckle waves can be obtained from

$$a = \frac{2\lambda}{\pi}\sqrt{\kappa(z - z_n)} \tag{2}$$

which relates the geometry of the buckled state with the applied curvature κ and the distance to the neutral axis $z - z_n$. The maximum strain in the fibers can then be obtained with:

$$|\epsilon_f| = \frac{d}{2}\kappa_f = \frac{d}{2}v'' = \frac{da\pi^2}{2\lambda^2} \tag{3}$$

Studies of elastic memory composites (Campbell et al., 2005; Schultz et al., 2007) and also of carbon–fiber composites with a silicone matrix (López Jiménez and Pellegrino, 2009) have extended Rosen's model and all assumed small strains and a linear constitutive relationship both for the fibers and the matrix.

Turning to the second topic, large strain formulations for fiber reinforced composites were investigated by Merodio and Ogden (2002, 2003). Their model, based on Spencer (1972), introduces two invariants to model the effect of the incompressible fibers on the strain energy of the material. Following Geymonat et al. (1993), the appearance of macroscopic instabilities is related to the loss of ellipticity of the homogenized constitutive relationships. Several biological materials, such as the cornea or blood vessels, fit the description of stiff fibers in a very soft matrix and models based on such an approach have been used by Holzapfel et al. (2000), Pandolfi and Manganiello (2006), Pandolfi and Holzapfel (2008). The second order homogenization theory developed by Ponte Castañeda (2002) has also been applied to fiber-reinforced hyperelastic materials. This approach is still under development, see for example Lopez-Pamies and Ponte Castañeda (2006), Agoras et al. (2009) and Lopez-Pamies and Idiart (2010); a comparison to numerical results has been presented in Moraleda et al. (2009b). In all of these studies the loading is two dimensional, usually under the assumption of plane strain. To the authors' knowledge, there is no detailed experimental or theoretical analysis of fiber reinforced soft materials under bending, which is the situation most relevant to the present study.

The final topic is strain softening, of a type known to occur in particle reinforced rubbers, such as tires. Their behavior is shown schematically in Fig. 2. If a virgin material sample is loaded to the strain level (1) it follows the path (a), known as the primary