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# Nonlinear hardening and softening resonances in micromechanical cantilever-nanotube systems originated from nanoscale geometric nonlinearities

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ABSTRACT

Micro/nanomechanical resonators often exhibit nonlinear behaviors due to their small size and their ease to realize relatively large amplitude oscillation. In this work, we design a nonlinear micromechanical cantilever system with intentionally integrated geometric nonlinearity realized through a nanotube coupling. Multiple scales analysis was applied to study the nonlinear dynamics which was compared favorably with experimental results. The geometrically positioned nanotube introduced nonlinearity efficiently into the otherwise linear micromechanical cantilever oscillator, evident from the acquired responses showing the representative hysteresis loop of a nonlinear dynamic system. It was further shown that a small change in the geometry parameters of the system produced a complete transition of the nonlinear behavior from hardening to softening resonance.

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## 1. Introduction

Aiming for a wide range of applications, particularly in sensing, signal processing, and fundamental research, micro- and nano-mechanical resonators have been extensively studied in the last decade (Ekinci and Roukes, 2005; Kim and Chun, 2007). While transition from linear to nonlinear resonance was found to occur readily at these size scales, the underlying nonlinear characteristics have been largely trivialized (Ekinci and Roukes, 2005) or considered detrimental (Ekinci et al., 2004; Ekinci and Roukes, 2005; Kozinsky et al., 2006; Kacem and Hentz, 2009; Kacem et al., 2010) to the design objectives during the early development stage. Over the past few years, however, nonlinear dynamics in resonating systems at micro/nanoscale has drawn great attention as researchers begin to learn to tailor system properties to achieve novel applications (Rhoads et al., 2005; Lifshitz and Cross, 2008; Stanton et al., 2010). For example, using a doubly-clamped carbon nanotube, a nonlinear nanoresonator with tunable and broad bandwidth was developed, and its nonlinear instability was applied for sensing external perturbations such as extremely small changes in mass and energy dissipation in ambient environments (Cho et al., 2010).

The most commonly encountered nonlinear behavior in micro/nano-beam resonators, especially with doubly-clamped ends, is hardening resonance originated from the involvement of tension induced during the oscillatory transverse motion of the beam (Husain et al., 2003; Cho et al., 2010). Often this geometrically nonlinear behavior is combined with external perturbations such as nonlinear potential fields (Touze et al., 2004; Kacem and Hentz, 2009; Kacem et al., 2010; Mestrom et al., 2010; Rhoads et al., 2010; Elshurafa et al., 2011) and thermal radiation (Sahai et al., 2007; Sahai, 2010) to induce and study complex nonlinear dynamics such as backbone transitions between hardening and softening resonances. However, to introduce intrinsic nonlinearity into a widely used cantilever type micro/nanoscale beam resonator where there is one free end to relax the internal tension, special design considerations are needed. In this study, nonlinear dynamics was realized in a micromechanical cantilever type system by incorporating a nanotube coupling to introduce geometric nonlinearity intentionally.

We first analytically study the micromechanical cantilever system integrated with geometric nonlinearity to determine its dynamic behavior, and then demonstrate through experiment the predicted nonlinear response. The intentional integration of strong nonlinearity into an otherwise linear cantilever resonator generates rich dynamics such as hardening or softening resonances, as well as nonlinear hysteresis loops leading to instability jumps. The general finding of this work is that strong intentional stiffness nonlinearity can be efficiently induced in the micro- and nanoscale systems by appropriate implementation of geometric or kinematic nonlinearities of a linearly elastic stiffness element. In addition, somewhat counter-intuitively, we demonstrate that the appropriate incorporation of even a nanoscale element (a boron

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nitride nanotube) is sufficient to induce strong nonlinearity in a linear microscale dynamic system many orders of magnitude larger in size. To our knowledge, this is one of the first works reporting on intentional strong nonlinearity induced in a microsystem by a nanoscale attachment.

## 2. Description and modeling

The scanning electron microscope (SEM) images of the representative nonlinear system investigated in this work are presented in Fig. 1. It consists of two micromechanical cantilevers bridged between two free ends by a multi-walled boron nitride nanotube (BNNT). Titled view at 52° in SEM in Fig. 1(b) reveals that the two cantilevers are nearly coplanar. The attached BNNT is ~2.6 μm in length and ~80 nm in diameter. The two microcantilevers are separately anchored to a solid base, so the only mechanical coupling is through the attached BNNT. The end segments of the BNNT are rigidly fixed onto the surfaces of the free ends of the microcantilevers through localized e-beam induced deposition of platinum. In the absence of the nanotube, there is no mechanism to introduce stiffness nonlinearity into these two cantilevers, and their dynamics is linear within the range of their operation. The two microcantilevers are designed with their first eigenfrequencies sufficiently apart so that when one of the microcantilevers oscillates near its fundamental resonant frequency, the other is stationary and acts as a nearly fixed anchor for the nanotube. The microcantilevers couples through the attached nanotube and the major (and only) source of nonlinearity in our system is thus the stiffness nonlinearities induced in the nanotube when stretched axially at every period of oscillation of the microcantilever. Note that the nanotube is elastically soft in the lateral direction but comparatively rigid in the axial direction. As shown in the zoomed-in image in Fig. 1b, the gap between the free ends of the cantilevers is designed to be narrow to accommodate a short nanotube attachment at the scale of several micrometers in order to maximize the induced geometric nonlinearity. Using a simple lumped parameter model, we describe in the following the forced resonance of this intentionally nonlinear system; i.e., when one of the microcantilevers oscillates with a dominant frequency equal to that of the applied harmonic excitation.

### 2.1. Equations of motion

Fig. 2 shows the schematic model of the system shown in Fig. 1. The microcantilever oscillating in its fundamental mode is modeled by the vertical linear harmonic oscillator with mass ($m$), vertical spring ($k_1$), and viscous damper ($c_1$), excited by a harmonic force ($F_o \cos \omega t$). The microcantilever in a non-resonant state is modeled as the rigid ground. The nanotube is modeled as a massless linear spring ($k_2$), which introduces geometric nonlinearity into the system as the mass $m$ displaces in the vertical direction. To be more general, considering that in a fabricated device there might exist a non-coplanar offset between the free ends of microcantilevers for attaching nanotube, we ascribe an initial offset (tilt) angle $\phi$ in the model for the attachment of the nanotube. It will be shown later that this angle plays an important role in the geometric nonlinearity induced in our system: depending on its value the system can produce either hardening or softening responses. Referring to the notation of Fig. 2, the length of the nanotube spring due to the vertical displacement of the mass ($m$) is $L_2' = \sqrt{y^2 + L_2^2 - 2yL\sin\varphi}$. The vertical component of the tension $T$ in the nanotube caused by the oscillation of the microcantilever is then $T\cos\theta = k_2(L_2' - L_2)(y - L_2\sin\varphi)/L_2'$. Performing the balance of vertical forces applied on the mass ($m$) provides the following equation of motion of the microcantilever system with the coupling nanotube:

$$m\ddot{y} + c_1\dot{y} + k_1 y + T\cos\theta = F_o\cos\omega t \tag{1}$$

where overdot denotes differentiation with respect to time ($t$). Expanding the expression for the tension in the nanotube in Taylor series for small vertical displacements yields:

$$T\cos\theta = \left(k_2\sin^2\varphi\right)y - \left(\frac{3k_2L_2\sin\varphi}{2}\right)\left(\frac{y}{L_2}\right)^2 + \left(\frac{k_2L_2}{2}\right)\left(\frac{y}{L_2}\right)^3 + \ldots, \quad \left|\frac{y}{L_2}\right| << 1 \tag{2}$$

Here, we assumed small motions and small offset, i.e., $\varphi \ll 1$. Substituting Eq. (2) into Eq. (1) yields

$$m\ddot{y} + c_1\dot{y} + \tilde{k}_1 y + \tilde{k}_2 y^2 + \tilde{k}_3 y^3 = F_o\cos\omega t \tag{3}$$

where

$$\tilde{k}_1 = k_1 + k_2\sin^2\varphi, \quad \tilde{k}_2 = -\frac{3k_2}{2L_2}\sin\varphi, \quad \tilde{k}_3 = \frac{k_2}{2L_2^2}. \tag{4}$$

As expected, the nanotube spring introduces the cubic nonlinearity into the system, regardless of the existence of the offset angle. However, the offset angle $\varphi$ contributes to both the quadratic and linear spring constants of the system. This means that depending on the

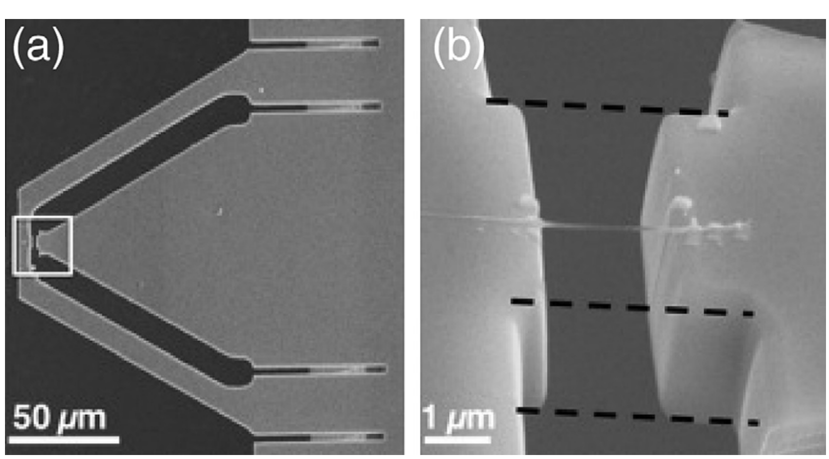


**Fig. 1.** SEM images of a nonlinear microcantilever system with integrated geometric nonlinearity through a nanotube attachment: (a) two micromechanical cantilevers are coupled through a nanotube, and (b) a magnified view of the marked region in square in (a) imaged at a sample tilt angle of 52°. The attached BNNT is ~2.6 μm in length and ~80 nm in diameter.

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