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A simple method for crack growth in mixed mode with X-FEM

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ABSTRACT

In this paper, a new method for level set update is proposed, in the context of crack propagation modeling with the extended finite element method (X-FEM) and level sets. Compared with the existing methods, such as the resolution of the Hamilton–Jacobi equations, this new method is much simpler because it does not required complex manipulations of the level sets. This method, called the "projection" method, uses both a classical discretization of the surface of the crack (segments for 2d cracks and triangles for 3d cracks) and a level set representation of the crack. This discretization is updated with respect to the position of the new crack front. Then the level sets are re-computed using the true distance to the new crack, by an orthogonal projection of each node of the structure onto the new crack surface. Then, numerical illustrations are given on 2d and 3d academic examples. Finally, three illustrations are given on 3d industrial applications.

1. Introduction

The issue of modeling crack propagation is a key aspect of many industrial studies. The simplest approaches are based on analytical formulas. But these codified methods are limited to very simple geometries and loading conditions. For more realistic configurations, the finite element method (FEM) is classically used. The main drawback is that the mesh must be updated at each propagation step, this task being hardly automatic for very complex geometries. Nevertheless, some attempts have been made to provide robust and reliable tools for automatic remeshing of 3d cracks (Dhondt, 1998; Maligno et al., 2010; Moslemi and Khoei, 2009; Schöllmann et al., 2003). Recently, OENRA team has made strong improvements of remeshing algorithms, leading to robust 3d crack propagations (Chiaruttini et al., 2011). Alternative methods to FEM and remeshing exist for modeling crack propagation, such as boundary integrals equations or the boundary element method (Citarella and Buchholz, 2008; Lucht, 2009) but they are less used than the FEM. All these methods lack of flexibility and the question of modeling crack topology changes (bifurcation, intersections, etc.) is still an open issue.

These difficulties explain the success of recent approaches in which the crack is not meshed: the extended finite element method (X-FEM) (Moës et al., 1999), the generalized finite element method (Duarte et al., 2001) and mesh-less methods (Duflot, 2006). One of the first paper on modeling 3d crack growth with an enriched approximation is due to Duarte et al. (2001). They introduce a crack representation with triangles. At the same time, the concept of level set has been introduced to represent an evolving 2d crack with the X-FEM (Stolarska et al., 2001). In the paper of Stolarska et al. (2001) the authors describe a methodology to represent a crack with two level set functions and give also a simple algorithm for modeling 2d crack growth: the level set functions are updated on a small region of elements surrounding each crack tip by a simple reconstruction of the true distance functions (with formulas using geometrical considerations). This technique has been reused and adapted later (Guidault et al., 2008; Ventura et al., 2003). The general framework to study non-planar 3d crack growth using X-FEM and the level set method leads to the Hamilton–Jacobi equations, which can be solved by a "simplex" procedure (Gravouil et al., 2002). The main difficulty is that two level sets are required to model a crack. This algorithm has been used widely since, for example for crack propagation in industrial structures (Bordas and Moran, 2006). The introduction of the Fast Marching Method has allowed one to solved 3d crack propagation (Chopp and Sulumar, 2003; Sukumar et al., 2003). Note that in these two papers, only pure mode I problems are treated. Efficiency can be improved when considering a structured mesh for the level set update, since on a regular mesh, a finite difference scheme can be directly used. Nevertheless, complex structures are often meshed with tetrahedrons. Adding an auxiliary regular grid to

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the global mesh circumvents this point (Prabel et al., 2007). With this approach, mechanical fields (displacement, stress, etc.) are computed on the whole mesh, and the level sets evolution equations are computed only on the regular grid. Recently, a review of several techniques for crack propagation with level sets has been made (Duflot, 2007). This paper is very interesting because the author compare the simple algorithm of Stolarska et al. (2001) and the method of Gravouil et al. (2002) on a crack in 2d propagation with a sharp kink. The author shows that if the J-integral is set on the final straight segment of the crack, all the methods give the same results. Differences appear when the size of the integral domain is larger, the method of Stolarska et al (2001) being better (but not optimal) than the approach of Gravouil et al. (2002). Duflot (2007) proposes several algorithms to retrieve level sets with good properties. Very recently, Colombo and Massin (2011) have proposed a robust method for high bifurcation angles.

The X-FEM has also been used to model cracks in the context of cohesive zones. Only the surface of discontinuity needs to be represented. The location of the crack front is numerically given by the values of the cohesive zone model. If level sets are used, only one level set is required. In Comi and Mariani (2007), de Borst et al. (2006), Mariani and Perego (2003), Unger et al. (2007), a simple discretization of the 1d crack with linear segment is made. Even if there is only a surface to be modeled, problems appear in 3d and numerical techniques proposed are complex. In Gasser and Holzapfel (2006), the authors propose a non-local tracking algorithm: the predictor step computes a discontinuity and the corrector step modifies the orientation of the discontinuity by a smoothing algorithm. A detailed comparison of most common 3d crack tracking algorithm is presented in Jäger et al. (2008), in which a global tracking seems to be the most general solution. Valance et al. (2008) describe a similar global tracking algorithm. The governing equations of level set (Hamilton–Jacobi equations) are then solved by a finite element technique. Another way to ensure the continuity of the crack discretization with level sets is presented in Duan et al. (2009). A local crack tracking algorithm is also proposed and applied in the context of a Partition of Unity enriched meshfree-method (Rabczuk et al., 2010). Their paper gives also an interesting overview of crack tracking algorithms in 3d.

The objective of the present paper is to introduce a simple method – called the "projection" method – to update the level sets in the X-FEM framework. The previous mentioned techniques to update the level sets, such as the resolution of the Hamilton–Jacobi equations, with or without an auxiliary grid, are very complex to implement in a robust way within a finite element code. Moreover, the crack position and the crack path are solely represented by the level sets. As a consequence, the visualization of the crack is not very easy and required plots of iso-zeros of the level sets. To visualize the crack front, intersections of iso-zeros are needed. Such operations are not very well handled by standard visualization tools. We propose in this paper a new method to update the level sets, which has two main advantages. The first advantage is an easier development, compared with the complexity of the resolution of the Hamilton–Jacobi equations. The second advantage is an easier visualization. Therefore, this new method is quite simple to use for industrial studies. In this method, we use both a classical discretization of the surface of the crack (segments for 2d cracks and triangles for 3d cracks) first introduced by Duarte et al. (2001) in the framework of G-FEM and a level set representation of the crack. This discretization is updated with a propagation criterion and a fatigue law. Then the level sets are re-computed using the true distance to the new crack, as in Stolarska et al. (2001). This technique is modified and extended for 3d cracks. The proposed method is also different from the vector level sets of Ventura et al. (2003) in the sense that Ventura do not use explicit representation of the crack surface, but store only the successive locations of the crack tip. The level sets are re-computed only with the knowledge of the current and the previous crack tip locations. It should be noted that this method is not really extendable in 3d. Moreover, this method also alleviates typical difficulties of remeshing algorithm. For example in Maligno et al. (2010), it is said that the simulation of a break-through failure with Zencrack is not possible and must be done manually. Such difficulties are easily done with the authors' method.

In Section 2, we present the theoretical aspects of the paper: the level set method (Section 2.1), the extended finite element method (Section 2.2) and the stress intensity factors evaluation (2.3). Section 3 is devoted to the description of the different techniques for the level set update. After having recalled the methods based on the resolution of the Hamilton–Jacobi equations (Section 3.3), we will focus on the "projection" method detailed in Section 3.4. Section 4 presents academical numerical examples to validate the proposal method on different 2d examples of crack propagation, in terms of crack paths. No significant differences are observed on the results between this new method and other classical update algorithms. Illustrations on 3d industrial studies are shown in Section 5, proving that this method can easily be used for engineering applications.

All the numerical methods presented for the level set update have been implemented in *Code_Aster*,[1] an industrial and open source finite element software developed by EDF. This software is also used for all the numerical studies carried out in Sections 4 and 5.

1.1. Remark

During the reviewing period of this paper, a very similar level set update algorithm has been developed by Fries and Baydoun independently of the authors' present paper. An on-line version of their paper is available (Fries and Baydoun, 2011). Their method uses also an explicit representation of the crack. Some differences exist, such as the number of level sets used: three level sets in Fries and Baydoun (2011) and two level sets in the present paper. By the way, it should be noted that the paper of Fries and Baydoun (2011) is very clear and interesting.

2. X-FEM for crack analysis

As the crack in not meshed with X-FEM, an additional information is needed to described the crack. The representation of the crack is then usually done with the level set method. This part describes briefly the essential theoretical aspects of the level set method, the extended finite element method, and the stress intensity factors evaluation.

2.1. The level sets method

Level sets are used to represent an evolving interface independently of the mesh. Basically, the level set function is the signed distance to the interface. Points where the level set is positive are "above" the interface, points where the level set is negative are "below" the interface and points where the level set is equal to zero are "on" the interface. To describe a crack, two level set functions are required (Stolarska et al., 2001). The normal level set (*lsn*) represents the distance to the crack surface (extended to the whole body) and the tangent level set represents the distance to the crack front. In this case, the level sets are real distance functions, chosen to be orthogonal on the crack surface. We underline the fact that the crack surface is given by $lsn(x) = 0 \cap lst(x) < 0$, and that the crack

[1] See www.code-aster.org.