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Inverse determination of liquid viscosity by means of the Bleustein–Gulyaev wave

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ABSTRACT

The Bleustein–Gulyaev (B–G) wave in piezoelectric materials of orthorhombic mm2 crystal class is reported to be a promising candidate for application to liquid sensing. In this work we present a rigorous quantitative investigation of the propagation of B–G wave in mm2 crystals in contact with a viscous liquid. An inversion algorithm is formulated to determine the liquid viscosity from the wave speed and attenuation data. Numerical results and discussions are given for potassium niobate ($KNbO_3$). The inversion results demonstrate that the liquid viscosity can be successfully determined from wave propagation characteristics.

1. Introduction

Surface acoustic wave (SAW) based sensors have been applied successfully within many technological fields, such as NDE (non-destructive examination) of materials, chemistry, biology and environment science. This is largely due to their superb sensitivity, speed and reliability (Hoummady et al., 1997; McMullan et al., 2000; Vellekoop, 1998; Lee et al., 2009). The development of micro-acoustic wave sensors in bio-sensing has created the need for further investigation of surface wave propagation in viscous liquid loaded piezoelectric structures (Wu and Wu, 2000). A number of acoustic wave modes have therefore been utilized for investigating various sensor applications. The influence of a viscous liquid on acoustic waves propagating in elastic or piezoelectric materials has been studied by several researchers; this is of particular interest for the development of liquid viscosity sensors (Zaitsev et al., 2001; Lee and Kuo, 2006; Zhang et al., 2001; Yang and Wang, 2008).

Zhang et al. (2001) proposed that the Bleustein–Gulyaev (B–G) wave in mm2 crystal class piezoelectric materials is a promising candidate for elucidating various characteristics in liquid sensing applications. The B–G wave does not radiate energy into the contacting liquid and is sensitive to changes in the liquid density and viscosity. However, Zhang et al. (2001) did not give a detailed quantitative analysis of the characteristics of the B–G wave propagating in piezoelectric materials loaded with viscous liquid. Guo and Sun (2008) derived the exact dispersion relation for a B–G wave propagating in a half-space composed of 6 mm piezoelectric material loaded with viscous liquid. Their results show that the so-called electrically shorted boundary condition is more suitable for liquid sensing applications than the open-circuit boundary condition. Later, Qian et al. (2010) studied the effect of thickness of the liquid layer on B–G wave propagation. We also remark that Du et al. (2010) investigated the properties of shear horizontal surface acoustic wave propagation in layered functionally graded piezoelectric structures loaded with viscous liquid.

Kielczyński and Plowiec, 1989 and Kielczyński et al., 2004 proposed the method of measurement of rheological properties of viscoelastic liquids using B–G wave and gave theoretical analyses of the influence of viscoelastic fluids on propagation of the B–G wave. They obtained the relations between the change in the complex propagation constant of the B–G wave and the shear acoustic impedance of the liquid by applying the theory of perturbation, which assumes that the liquid does not significantly modify the properties of the acoustic waves. Later, they also applied their method to measure liquid viscosity at high pressure for various temperatures (Kielczyński et al., 2011).

Piezoelectric mm2 crystals support B–G waves and offer promising substrates for liquid sensing applications (Zhang et al., 2001; Royer and Dieulesaint, 2000). In particular, one example of the mm2 crystal piezoelectric material $KNbO_3$, which exhibits a higher electro-mechanical coupling factor, offers particularly good potential for liquid sensing applications. Although a lot of work has been done in respect of the application of surface waves in piezoelectric media to liquid sensing for various scenarios, little work is available in the literature on the inversion algorithm of determining liquid properties from wave propagation parameters. This is of utmost importance for implementation of liquid sensing by means of the surface acoustic wave. In this present study, we will present a rigous quantitative investigation of the propagation of B–G waves

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in mm2 crystals in contact with viscous liquid. The dispersion relation for metalized surface boundary conditions is obtained. Also, we study the inversion algorithm for determination of liquid viscosity from wave propagation characteristics. Numerical results of attenuation and phase velocity against viscosity, density of the liquid and wave frequency are given for potassium niobate ($KNbO_3$). Results for different kinds of error functions are also compared. The results of this study are expected to provide useful data and guidelines for liquid senor design and development.

2. Description of the problem

The problem considered is concerned with a shear type surface wave propagating in mm2 piezoelectric material in contact with viscous liquid, as shown in Fig. 1. The piezoelectric material occupies the half-space $x_2 < 0$ and the liquid covers the half-space $x_2 > 0$, with x_3 the axis parallel to the twofold axis of symmetry. The x_1 and x_2 axes are parallel to the X and Y axes of the crystallographic coordinates (XYZ), respectively. This configuration is called Y cut-X propagation, which exhibits the maximum electromechanical coupling factor (Nakamura and Oshiki, 1997).

In the absence of body force and free electric charge, the coupled electroelastic governing equations for piezoelectric media can be written as (Zhang et al., 2001)

$$\begin{cases} C_{ijkl}u_{k,jl} + e_{kij}\phi_{,kj} = \rho\frac{\partial^2 u_i}{\partial t^2} \\ e_{jkl}u_{k,jl} - \varepsilon_{ik}\phi_{,ki} = 0 \end{cases} \quad (i,j,k,l = 1,2,3). \tag{1}$$

The elastic constants C_{ijkl} can be written into contracted form $C_{\alpha\beta}$ by the following rule

$$\begin{aligned} &\alpha = 9 - i - j, \quad \beta = 9 - k - l \quad \text{if } i \neq j, \quad k \neq l, \\ &\alpha = i = j, \quad \beta = k = l \quad \text{if } i = j, \quad k = l. \end{aligned} \tag{2}$$

Similarly, the piezoelectric constants e_{ijk} can also be put into contracted form $e_{i\alpha}$ with the index α observing the rule outlined in Eq. (2).

For mm2 piezoelectric materials, with x_3 direction being the 2-fold symmetry axis, Eq. (1) takes the following form

$$\begin{aligned} &C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + C_{55}\frac{\partial^2 u_1}{\partial x_3^2} + (C_{12}+C_{66})\frac{\partial^2 u_2}{\partial x_1\partial x_2} \\ &\quad + (C_{13}+C_{55})\frac{\partial^2 u_3}{\partial x_1\partial x_3} + (e_{31}+e_{15})\frac{\partial^2 \phi}{\partial x_1\partial x_3} = \rho\frac{\partial^2 u_1}{\partial t^2}, \\ &(C_{11}+C_{66})\frac{\partial^2 u_1}{\partial x_1\partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} + C_{44}\frac{\partial^2 u_2}{\partial x_3^2} \\ &\quad + (C_{23}+C_{44})\frac{\partial^2 u_3}{\partial x_2\partial x_3} + (e_{32}+e_{24})\frac{\partial^2 \phi}{\partial x_2\partial x_3} = \rho\frac{\partial^2 u_2}{\partial t^2}, \\ &(C_{13}+C_{55})\frac{\partial^2 u_1}{\partial x_1\partial x_3} + (C_{23}+C_{44})\frac{\partial^2 u_2}{\partial x_2\partial x_3} + C_{55}\frac{\partial^2 u_3}{\partial x_1^2} \\ &\quad + C_{44}\frac{\partial^2 u_3}{\partial x_2^2} + C_{33}\frac{\partial^2 u_3}{\partial x_3^2} + e_{15}\frac{\partial^2 \phi}{\partial x_1^2} + e_{24}\frac{\partial^2 \phi}{\partial x_2^2} + e_{33}\frac{\partial^2 \phi}{\partial x_3^2} = \rho\frac{\partial^2 u_3}{\partial t^2}, \\ &(e_{15}+e_{31})\frac{\partial^2 u_1}{\partial x_1\partial x_3} + (e_{24}+e_{32})\frac{\partial^2 u_2}{\partial x_2\partial x_3} + e_{15}\frac{\partial^2 u_3}{\partial x_1^2} + e_{24}\frac{\partial^2 u_3}{\partial x_2^2} \\ &\quad + e_{24}\frac{\partial^2 u_3}{\partial x_3^2} - \varepsilon_{11}\frac{\partial^2 \phi}{\partial x_1^2} - \varepsilon_{22}\frac{\partial^2 \phi}{\partial x_2^2} - \varepsilon_{33}\frac{\partial^2 \phi}{\partial x_3^2} = 0. \end{aligned} \tag{3}$$

Now consider a harmonic wave propagating in the x_1 direction, with all physical quantities only dependent on the in-plane variables (x_1, x_2), and independent of x_3. This case is an example of a generalized plane strain problem. In this situation, Eq. (3) is further simplified into

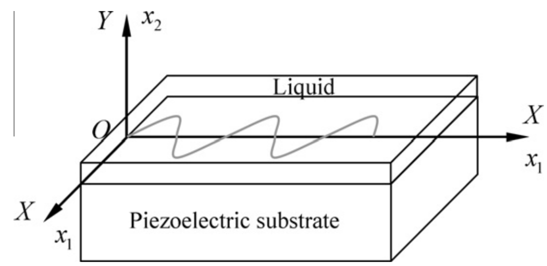

Fig. 1. A piezoelectric substrate in contact with viscous liquid.

$$\begin{aligned} &C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + (C_{12}+C_{66})\frac{\partial^2 u_2}{\partial x_1\partial x_2} = \rho\frac{\partial^2 u_1}{\partial t^2}, \\ &(C_{12}+C_{66})\frac{\partial^2 u_1}{\partial x_1\partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} = \rho\frac{\partial^2 u_2}{\partial t^2}, \\ &C_{55}\frac{\partial^2 u_3}{\partial x_1^2} + C_{44}\frac{\partial^2 u_3}{\partial x_2^2} + e_{15}\frac{\partial^2 \phi}{\partial x_1^2} + e_{24}\frac{\partial^2 \phi}{\partial x_2^2} = \rho\frac{\partial^2 u_3}{\partial t^2}, \\ &e_{15}\frac{\partial^2 u_3}{\partial x_1^2} + e_{24}\frac{\partial^2 u_3}{\partial x_2^2} - \varepsilon_{11}\frac{\partial^2 \phi}{\partial x_1^2} - \varepsilon_{22}\frac{\partial^2 \phi}{\partial x_2^2} = 0. \end{aligned} \tag{4}$$

Similar to the case of piezoelectric materials with 6 mm symmetry (Guo and Sun), it can be seen from Eq. (4) that (u_1, u_2) is decoupled from (u_3, ϕ). The first two equations of Eq. (4) show that (u_1, u_2) may constitute a purely elastic Rayleigh wave, whereas the last two equations indicate that (u_3, ϕ) could comprise a shear type surface electroelastic wave, a wave more commonly known as the Bleustein–Gulyaev wave.

The liquid is assumed to be a viscous Newtonian fluid. Suppose the motion of the liquid is induced only by wave propagation in the piezoelectric material and also propagates in the form of a harmonic wave. In regard to this problem, the embroil inertial term in the Navier–Stokes equation can be omitted. Moreover, the pressure gradient can also be ignored since only shear deformation occurs during wave propagation (Guo and Sun, 2008). Therefore, the governing equation for the liquid is simplified to

$$\frac{\partial v_3}{\partial t} - \frac{\mu_l}{\rho_l}\nabla^2 v_3 = 0, \tag{5}$$

where ρ_l is the mass density of the liquid, μ_l the dynamic viscous coefficient of the liquid and v_3 is the liquid particle velocity in the x_3 direction.

3. Dispersion relation

For this wave propagation problem, the displacement component u_3 and electric potential ϕ can be assumed to take the following form

$$\begin{aligned} u_3 &= W(x_2)e^{ik(x_1 - vt)} = W(x_2)e^{i(kx_1 - \omega t)}, \\ \phi &= \Phi(x_2)e^{ik(x_1 - vt)} = \Phi(x_2)e^{i(kx_1 - \omega t)}, \end{aligned} \tag{6}$$

where k is wave number, v phase velocity of the wave, ω angular frequency, i is the imaginary unit, and $W(x_2)$ and $\Phi(x_2)$ are unknown functions of x_2.

Substituting the above expressions for u_3 and ϕ into the last two equations of Eq. (4) leads to

$$\begin{cases} C_{44}W''(x_2) + k^2(\rho v^2 - C_{55})W(x_2) - k^2 e_{15}\Phi(x_2) + e_{24}\Phi''(x_2) = 0, \\ e_{24}W''(x_2) - k^2 e_{15}W(x_2) + k^2\varepsilon_{11}\Phi(x_2) - \varepsilon_{22}\Phi''(x_2) = 0, \end{cases} \tag{7}$$

where the superscript prime denotes differentiation with respect to the variable x_2.