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Analysis of micro/nanobridge test based on nonlocal elasticity

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ABSTRACT

Based on the nonlocal Timoshenko beam theory, we develop a mechanics approach to analyze the micro/nanobridge test. This approach considers the shear deformation, the strain gradients, the substrate deformation, and the contact deformation between the indenter bar tip and a tested beam, resulting in an analytic solution of beam deflection versus applied load involving other parameters of material intrinsic length, film residual stress, and cylinder bar radius. The same approach was further developed to analyze the delamination test, giving explicit formulas for the energy release rate and the phase angle.

1. Introduction

Testing and characterizing the mechanical properties of low dimensional materials, such as nanowires, nanotubes, and micro/nano thin films, are drawing much attentions due to the wide applications of these materials in micro/nanoelectromechanical systems (M/NEMS) and micro/nanoelectronic devices. Various mechanical characterization techniques have been developed (Volinsky et al., 2002; Yang and Li, 2008), among which bending test is one of the simplest and most important experimental methods. For example, Weihs et al. (1988) conducted cantilever beam bending tests to characterize the strength and stiffness of thin film materials. With such tests, Wong et al. (1997) measured the Young's modulus of nanorods and nanotubes. Poncharal et al. (1999) proposed a cantilever beam resonance method to determine the Young's modulus of carbon nanotubes. Nam et al. (2006) also used this method to characterize the electromechanical properties of GaN nanowires.

In both static and dynamic theoretical studies, the clamped boundary condition at the support end of a beam is presumed, implying that the supporting substrate is treated to be mechanically rigid. In fact, however, the supporting substrate deforms elastically as the beam bends. Baker and Nix (1994) found that the substrate deformation has a significant effect on the results from cantilever beam-bending tests. Zhang et al. (2000a) theoretically proved Baker and Nix's formula of the load–displacement relation by analyzing substrate deformation with two coupled linear springs. Zeng and Zheng (2007) systematically studied the effects of various clamp uncertainties (partially pinned, elastic foundation, unknown defects, etc.) on the Young's modulus determined from the dynamic resonance test and showed that those effects cannot be ignored even at the nanometer scale.

In addition to cantilever beam bending tests, another important type of beam bending test is called the micro/nanobridge test, in which both ends of a micro/nanobeam are bonded onto a substrate. Micro/nanobridge samples are usually fabricated with film/substrate systems by using the micro/nanoelectronic fabrication technique. Residual stress in the tested thin film, if any, cannot be released along the beam length direction so that the micro/nanobridge test is capable to characterize simultaneously the elastic constant, residual stress and bending fracture strength of brittle materials (Kobrinsky et al., 2000; Zhang et al., 2000b). The micro/nanobridge test was first developed for single-layer thin films (Kobrinsky et al., 2000; Zhang et al., 2000b) and the substrate deformation was modeled by using three coupled linear springs. The derived analytic load-deflection relationship is easily used in practice. For ductile materials, the micro/nanobridge test is also able to characterize the plastic properties of the tested beams besides the elastic properties (Huang and Zhang, 2006). Furthermore, the micro/nanobriedge test has been further developed to characterize the mechanical properties of bilayer, trilayer and multilayer thin films (Su et al., 2000; Xu and Zhang, 2003; Wang et al., 2005; Zhang et al., 2005a; Wang et al., 2006b; Wang and Zhang, 2007),

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and nanowires (Chan et al., 2010). The review article (Zhang, 2008) gives a detailed description of the micro/nanobridge test and interested readers may consult it.

If the external load is applied from the backside of the tested beam, interfacial cracking may occur at the two ends along the interface between the tested beam and the substrate. In this case, the micro/nanobridge test is converted into the delamination test. The delamination test can characterize the interfacial fracture behavior of a film/substrate system.

Moreover, the size-dependent properties of materials have been observed in tests on micro/nanometer sized samples, structures and systems. For example, Nam et al. (2006) found the diameter-dependent electromechanical properties for GaN nanowires. Puri et al. (2009) reported the size-dependent yield strength. The theoretical analysis of the above mentioned beam bending tests is based on the classical beam theory, which is independent of length-scale and, of course, cannot academically investigate the size effect. Eringen and his co-workers (Eringen, 1972, 1983; Eringen and Edelen, 1972) developed the nonlocal elasticity theory. In recent years, this theory has been successfully applied in explaining some size-dependent effects, such as the deformation of beam structures (Peddieson et al., 2003), buckling and vibration of multiwalled carbon nanotubes (Sudak, 2003; Zhang et al., 2004; Zhang et al., 2005b; Wang and Hu, 2005), and so on. Moreover, a theoretical point load, which causes stress singularity, is an approximation and simplification to real cases. This simplification works at the macroscopic scale, but may cause errors at the nanometer scales, at which most tips have a spherical shape and the tip radius may change from tip to tip. Wan and Liao (1999) adopted the blunted tip approach to determine the mechanical properties of the thin flexible membrane. Considering all aspects mentioned above, we are motivated to develop, in the present paper, a nonlocal mechanics theory for the micro/nanobridge test with a cylinder bar indenter of a finite radius. If the contact between the cylinder bar and a tested beam was analyzed with the Euler–Bernoulli or the simplest beam theory as in Timoshenko (1957), the curvature of the beam in contact with the cylinder would be constant and thus, the moment in this part of beam would be constant, leading to a zero shear force there and no contact force in the contact part. The reason to cause the unrealistic solution is the restriction of the shear deformation. Indeed, the effect of the shear deformation is important and cannot be neglected, especially for stocky beam on elastic substrate (Essenburg 1960; Zhang et al. 2006). High-order beam theory (Timoshenko, 1921) must be used in these cases. Therefore, we analyze in Section 2 mechanics underlying the micro/nanobridge test based on the nonlocal Timoshenko beam theory. For simplicity, we perform a two-dimensional analysis with a cylinder bar of radius R providing the mechanical load. This analysis, described in Section 3, leads to the general solution. In Section 4, we present a final analytical solution for the micro/nanobridge test. The results are illustrated and discussed in Section 5. In Section 6, we study the energy release rate and the phase angle for the delamination test. Concluding remarks are given in Section 7.

2. Nonlocal theoretical analysis of the micro/nanobridge test

Fig. 1 shows a schematic of the micro/nanobridge test, where a cylindrical indenter of radius R applies mechanical load to a tested beam of thickness h and half-length a, the two ends of the beam are bonded onto a substrate. The coordinate system oxy is set up with x-axis along the middle plane of the tested beam before loading and y-axis directed down through the center of the cylinder, as shown in Fig. 1. Assuming symmetrical loading, only the left half of the beam $-a < x < 0$ is analyzed.

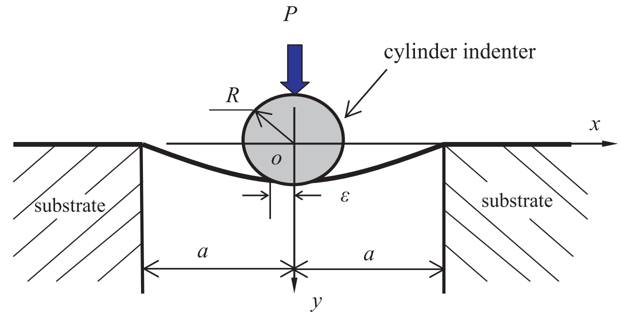

Fig. 1. Schematic of the bridge test with a cylinder indenter subjecting a load to a beam, where x-axis is on the middle plane of the tested beam before loading and y-axis directs down through the center of the cylinder.

Under deformation, the beam-tip contact zone is $-\varepsilon < x \leqslant 0$, leaving the $-a < x < \varepsilon$ region contact-free. In studying the contact between beam and cylinder, the nonlocal Timoshenko beam theory is used, and the displacement along the y direction, w, and the rotation angle, φ, are determined by two governing equations of Wang et al. (2006a):

$$C\frac{d^2w}{dx^2} - C\frac{d\varphi}{dx} + N\frac{d^2w}{dx^2} = 0, \tag{1a}$$

$$D\frac{d^2\varphi}{dx^2} + C\frac{dw}{dx} - C\varphi + (el)^2N\frac{d^3w}{dx^3} = 0, \tag{1b}$$

where D is the bending rigidity, C is the shearing rigidity given by $C = k_f\frac{E_f}{2(1+\nu)}S$ with $k_f = \frac{10(1+\nu)}{12+11\nu}$ the rectangular cross-section coefficient and S the cross-sectional area of the beam, and N is the axial force. E_f and ν are respectively the Young's modulus and Poisson's ratio of the tested material. l is the material intrinsic length and e a constant (Eringen, 1983; Sudak, 2003). For a rectangular beam cross-section of unit width, the bending rigidity can be expressed by $D = \frac{E_f h^3}{12(1-\nu^2)}$.

Following Wang et al.'s approach (Wang et al., 2006a), we have the moment and shearing force in the bending material, which are expressed as

$$M = D\frac{d\varphi}{dx} - (el)^2N\frac{d^2w}{dx^2}, \quad Q = C(\frac{dw}{dx} - \varphi). \tag{2}$$

3. General solution

In the contact region, based on the geometrical relations of cylindrical indenter, the deflection can be expressed as

$$w_c = (w_0 - R) + \sqrt{R^2 - x^2}, \tag{3}$$

where w_0 is the deflection at $x = 0$, and the subscript "c" indicates contact. Substituting the Taylor series of deflection in Eq. (3) into Eq. (1b), one obtains the approximation

$$\frac{d^2\varphi_c}{dx^2} - \frac{C}{D}\varphi_c = \frac{t}{D}x + \frac{b}{D}x^3, \tag{4}$$

where

$$t = \frac{CR^2 + 3(el)^2N}{R^3}, \quad b = \frac{CR^2 + 15(el)^2N}{2R^5}. \tag{5}$$

In deriving Eq. (4), only the first two terms of the expanded series are taken. The accuracy will be verified later by finite element analysis.