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Cracked elastic layer under compressive mechanical loads

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ABSTRACT

We consider boundary value problem in which an elastic layer containing a finite length crack is under compressive loading. The crack is parallel to the layer surfaces and the contact between crack surfaces are either frictionless or with adhesive friction or Coulomb friction.

Based on fourier integral transformation techniques the solution of the formulated problems is reduced to the solution of a singular integral equation, then, using Chebyshev's orthogonal polynomials, to an infinite system of linear algebraic equations. The regularity of these equations is established. The expressions for stress and displacement components in the elastic layer are presented. Based on the developed analytical algorithm, extensive numerical investigations have been conducted.

The results of these investigations are illustrated graphically, exposing some novel qualitative and quantitative knowledge about the stress field in the cracked layer and their dependence on geometric and applied loading parameters. It can be seen from this study that the crack tip stress field has a mode II type singularity.

1. Introduction

Stress concentration is often a critical concern because it affects the durability and reliability of structures and their components. Stress concentrators in structures can exist as a result of material composition imperfections (cavities, inclusions) or they can be caused by technological and structural needs (holes, cuts, etc.). In either case, analyzing the effects of stress concentrators is very important.

Stress concentrators in the form of cracks have been intensively studied in the literature. Since experimental observations indicate that crack growth is often in the form of opening mode crack growth instead of mixed mode or pure shear mode crack growth, the research reported in the literature on the subject of crack growth has mainly focused on mode I fracture (e.g. Erdogan and Sih, 1963; Sih, 1974; Bilby and Cardew, 1975; Cotterell and Rice, 1980; Hayashi and Nemat-Nasser, 1981; or Broberg, 1987).

In the majority of works in the literature it is assumed that the crack surfaces are not in contact. However crack surface contact can occur under compressive loading and such cracks can pose a potential risk just as cracks under tensile loading (e.g. Roy et al., 1999; Deng, 1993, 1995; Dhirendra and Narasimhan, 1998; Ghonem and Kalousek, 1988; Hallbfack, 1998; Hayashi and Nemat-Nasser, 1981; Hearle and Johnson, 1985; Isaksson and Stahle, 2002, 2003; Ishida and Abe, 1996; Hancock, 1999; Makaryan, 2006; Makaryan et al., 2009; Melin, 1986). Due to the elimination of crack surface opening, the growth of cracks with crack surface contact is in the shear mode (or mode II under in-plane loading conditions).

El-Borgi et al. (2004) considered the problem of a functionally graded coating bonded to a semi-infinite homogeneous medium with a crack embedded in the FGM layer and parallel to the free surface. The composite medium is subjected to a frictional Hertzian contact traction loading applied to the surface of the graded coating. The author's utilize a crack closure algorithm whenever the mode I stress intensity factors turn out to be negative under the action of compressive loads.

Broberg (1987) reported laboratory produced mode II crack growth in plates in experiments conducted in a combination of pressure and shear loads. Hearle and Johnson (1985) achieved shear crack growth in experiments performed on rail steels subjected to a moving point load. Ishida and Abe (1996) carried out rolling contact tests in a rail/wheel contact fatigue testing machine and reported sub-surface crack growth in mode II. More recently, the propagation of cracks parallel with a shear loaded surface (which are sub-surface horizontal cracks) due to surface traction caused by contact, have been analytically and numerically investigated by several researchers (e.g. Wong et al., 1996; Jayaraman et al., 1997, Komvopoulos and Cho, 1997). Melin (1987) concluded that mode II crack growth in an elastic material would be preferred over mode I only if the ratio between the critical stress intensity factors K_{IIc} and K_{Ic} is fairly low. The effect of crack surface friction

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on mode II stress intensity factor of a central slant crack in a plate uniformly loaded in uniaxial compression is discussed by Hammouda et al. (2002). Comparison of predictions by mode II or mode III criteria on crack front twisting in three or four point bending experiments was done by Lazarus et al. (2008).

Problems with cracks under compressive forces can be interpreted as problems with contact of two separate bodies pressed against each other (El-Borgi et al., 2006; Keer et al., 1972). In these problems, the length of the contact zone and the contact pressure (which is zero at the ends of the contact segment) are the primary unknowns of the problem.

Practical applications where mode II crack propagation generally is predicted to prevail are in various applications subjected to a load combination of a shear stress and a high compressive normal stress (e.g. gear drives, rolling bearings, railway applications or structures exposed to earth quakes). However, despite the great number of investigations, the phenomena of crack initiation and propagation associated with compressive forces are still not fully understood.

The large number of available investigations in the literature (e.g. Broberg (1999), Civelek and Erdogan (1974), Hill (1950), Hutchinson (1968), Isaksson and Stahle (2002), Liu (1974), Sih et al. (1966), Sundara Raja Iyengar et al. (1988)) has been mainly restricted to the non-interacting crack surfaces (i.e. non-contacting crack surfaces).

In this paper we show that if an elastic layer containing a crack parallel to the layer boundary surfaces is under compressive loading on the layer boundary surfaces the shear stress can became infinite at the crack tips and at the same time the normal stress is finite along the crack line. In other words we show that compressive loading normal to the crack surfaces can lead to a mode II type stress singularity at the crack tips and can initiate mode II type crack growth. To that end, the plane problem of an elastic layer, weakened by a finite-length crack under compressive loading is considered, in which the crack surfaces are in full contact with either no friction or with Coulomb friction or a friction given in advance (i.e. adhesive friction).

Based on the Fourier integral transformation techniques the solution of the formulated problem is reduced to the solution of a singular integral equation, and then, using the Chebyshev's orthogonal polynomials, to an infinite system of linear algebraic equations. The regularity of these equations is established. These equations are solved numerically for several typical cases and the resulting stress distributions are described.

2. Problem description and formulation

Let us suppose that an elastic layer occupies the domain $-\infty < x < \infty$, $-h_1 \leqslant y \leqslant h_2$ and a finite crack $-x_0 < x < x_0$ is located at $y = 0$ (see Fig. 1). The layer surfaces $y = h_1$ and $y = -h_2$ are under

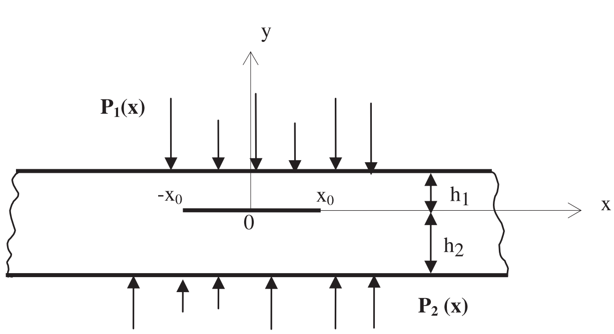

Fig. 1. An elastic layer weakened by a finite length crack and is under the action of compressive surface forces.

the action of certain compressive mechanical forces as shown in Fig. 1., and the crack surfaces are pressed together so that they are in full contact. In other words along the crack surfaces the normal stress satisfy the condition $\sigma_y(x,\pm 0) < 0$.

This elasticity plane problem can be reduced to solving the biharmonic equation below in Terms of an Airy stress function (Sneddon and Berry, 1958):

$$\Delta^2 \Phi(x,y) = 0. \tag{1}$$

The stress and displacement's components can be expressed through the function $\Phi(x,y)$ as:

$$\sigma_y(x,y) = \frac{\partial^2 \Phi(x,y)}{\partial x^2}, \quad \sigma_x(x,y) = \frac{\partial^2 \Phi(x,y)}{\partial y^2},$$

$$\tau_{xy}(x,y) = -\frac{\partial^2 \Phi(x,y)}{\partial x \partial y}, \tag{2}$$

$$u_x(x,y) = \frac{1}{E}\left(\int \frac{\partial^2 \Phi(x,y)}{\partial y^2} dx - \nu \frac{\partial \Phi(x,y)}{\partial x} + U_0\right), \tag{3}$$

$$u_y(x,y) = \frac{1}{E}\left(\int \frac{\partial^2 \Phi(x,y)}{\partial x^2} dy - \nu \frac{\partial \Phi(x,y)}{\partial y} + V_0\right). \tag{4}$$

In the above

$$\Delta^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{5}$$

is the Laplace operator; E and ν are the Young's modulus and Poisson's ratio, respectively; and

U_0 and V_0 are constants. Eqs. (1)–(4) are valid for a plane stress problem. For a plane strain problem E and ν should be replaced by $E \to \frac{E}{1-\nu^2}$ and $\nu \to \frac{\nu}{1-\nu}$. To avoid confusion, the Airy stress function and the associated stress and displacement quantities for the regions above and below the crack plane will be denoted by a subscript "1" or "2". Specifically, the region above the crack plane $-\infty < x < \infty$, $0 < y \leqslant h_1$ will be assigned the subscript "1", and the region below the crack plane $-\infty < x < \infty$, $h_2 \leqslant y < 0$ will be assigned the subscript "2".

2.1. Boundary conditions

The boundary conditions will be written in the following form:

$$\sigma_{\boldsymbol{y}}^{(j)}(\boldsymbol{x}, \boldsymbol{l_j}) = \boldsymbol{p_j}(\boldsymbol{x}), \quad \tau_{\boldsymbol{xy}}^{(j)}(\boldsymbol{x}, \boldsymbol{l_j}) = 0, \quad -\infty \leqslant \boldsymbol{x} < \infty;$$
$$j = 1,2; \quad l_1 = h_1, \quad l_2 = -h_2, \tag{6}$$

$$\sigma_y^{(l)}(x,0) = \sigma_y^{(2)}(x,0), \quad \tau_{xy}^{(l)}(x,0) = \tau_{xy}^{(2)}(x,0),$$
$$u_y^{(l)}(x,0) = u_y^{(2)}(x,0), \quad -\infty \leqslant x < \infty, \tag{7}$$

$$\begin{cases} u_x^{(1)}(x,0) = u_x^{(2)}(x,0) & x_0 < |x| < \infty, \\ \tau_{xy}^{(1)}(x,0) = \tau_{xy}^{(2)}(x,0) = \tau(x) & -x_0 < x < x_0. \end{cases} \tag{8a, b}$$

In addition to (6), (7), (8a,b) we should add the overall equilibrium condition of the layer

$$\int_{-\infty}^{\infty} x^k p_1(x) d\lambda = \int_{-\infty}^{\infty} (-1)^k x^k p_2(x) d\lambda, \quad k = 0,1 \tag{9}$$

In (8) the function $\tau(x)$ represents the distribution of shear stress on the contacting crack surfaces. In this paper for $\tau(x)$ we will assume:

$$\tau(x) = \begin{cases} 0 & \textit{for frictionless contact between crack surfaces}, \\ k_0 \sigma_y(x,0) & \textit{for Coulomb friction between crack surfaces}, \\ \tau_0(x) & \textit{for adhesive friction with a known shear stress}. \end{cases} \tag{10}$$

where k_0 is a Coulomb friction coefficient.