International Journal of Solids and Structures 48 (2011) 3312–3325

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A material model for cementitious composite materials with an exterior point Eshelby microcrack initiation criterion

I.C. Mihai *, A.D. Jefferson

Cardiff University, School of Engineering, Queen's Buildings, The Parade, CF24 3AA, UK

ARTICLE INFO

Article history:
Received 17 August 2010
Received in revised form 12 May 2011
Available online 12 August 2011

Keywords:
Micromechanics
Microcrack
Concrete
Constitutive
Interface

ABSTRACT

A micromechanical model for cementitious composite materials is described in which microcrack initiation, in the interfacial transition zone between aggregate particles and cement matrix, is governed by an exterior-point Eshelby solution. The model assumes a two-phase elastic composite, derived from an Eshelby solution and the Mori–Tanaka homogenization method, to which circular microcracks are added. A multi-component rough crack contact model is employed to simulate normal and shear behaviour of rough microcrack surfaces. The development of the microcrack initiation criterion and the rules adopted for microcrack evolution are a particular focus of the paper. Finally, it is shown, on the basis of several numerical simulations, that the model captures key characteristics of the behaviour of cementitious composites such as concrete.

1. Introduction

Extensive research has been carried out over the last few decades in order to explain and model damage phenomena in quasi-brittle materials such as concrete. It is generally accepted that the heterogeneous structure of such materials observed at micro and meso levels determines their complex macroscopic behaviour and failure mechanisms (van Mier, 1997).

A number of macroscopic phenomenological models based on damage and plasticity theories have been formulated (e.g. Comi and Perego, 2001; di Prisco and Mazars, 1996; Este and Willam, 1994; Feenstra and de Borst, 1995; Lee and Fenves, 1998; Grassl et al., 2002; Grassl and Jirásek, 2006; Nguyen and Korsunsky, 2008), in which the plastic and damage internal variables have been assumed to be scalars, vectors or higher order tensors. Although their numerical implementation in finite element codes is sometimes relatively straightforward, they do not, in general, properly capture all of the physical mechanisms that control the complex behaviour of these materials and can often use parameters that are difficult to determine and which do not have physical meanings.

In recent years, several models that aim to capture macroscopic behaviour by simulating the physical mechanisms at micro and meso levels have been developed using a micromechanical approach. Pensée et al. (2002) and Pensée and Kondo (2003) formulated an anisotropic damage model by employing a micromechanical solution for an elastic solid containing non-interacting penny-shaped microcracks and an energy release rate-based damage criterion that incorporates frictionless crack closure. Gambarotta (2004) proposed an anisotropic friction-damage model based on the solution of an elastic body containing plane cracks. Pichler et al. (2007) combined fracture energy theory and continuum micromechanics to formulate a damage evolution law in a tensile strain-softening model for brittle materials based on the propagation of interacting microcracks. Recently, Zhu et al. (2008, 2009) developed an anisotropic damage model using the classic Eshelby inclusion solution and a thermodynamics-based damage evolution law coupled with Coulomb friction sliding along closed crack surfaces. Unilateral effects as well as the interaction, shape and spatial distribution of microcracks were taken into account through a homogenization procedure based on the scheme proposed by Ponte-Castaneda and Willis (1995).

Micromechanical solutions have also been employed to develop effective models for the prediction of elastic and strength properties of cementitious materials (Pichler and Hellmich, 2011).

The Microplane model was originally inspired by micromechanics (Bazant and Prat, 1988) but was subsequently developed along a more phenomenological path (Bazant and Caner, 2005) and thus differs from the more mechanistic micromechanical models discussed above.

Jefferson and Bennett (2007, 2010) developed a micromechanical model that simulates a two-phase composite material containing randomly distributed penny-shaped microcracks which develop according to a local damage evolution function. The Mori–Tanaka homogenization method was adopted to account for the interaction between microcracks. Stress recovery across

* Corresponding author. Tel.: +44 (0) 29 20876043.
E-mail address: MihaiIC1@cf.ac.uk (I.C. Mihai).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2011.08.001

cracks that regain contact was enabled through a rough crack contact component introduced into the model.

The primary purpose of the work reported in this paper is to develop a constitutive model in which mechanisms described by micro-mechanical solutions are employed to predict the characteristic response of particulate cementitious materials. The work at this stage is concerned only with mature properties and mechanical behaviour. The model employs the essential ideas described in Jefferson and Bennett (2007, 2010), however the present model adopts a more mechanistic approach in that a number of phenomenological aspects of the previous model have been replaced with mechanistic components. The particular focus of the present contribution is the use of the exterior point Eshelby (EPE) solution to compute stress concentrations in the interfacial transition zones (ITZ) around inclusions. This is subsequently used to develop a microcrack initiation criterion and evolution function. In essence, the exterior point Eshelby tensor describes the disturbance in a stress or a strain field created by an ellipsoidal (in this case spherical) inclusion within the surrounding matrix phase.

This article is structured as follows:

- Section 2 gives the model theory which has 5 essential components as follows;
  - Section 2.1: two phase composite solution for elastic matrix and spherical inclusions.
  - Section 2.2: added compliance due to microcracks.
  - Section 2.3: exterior point Eshelby (EPE) solution.
  - Section 2.4: stress recovery through rough crack contact.
  - Section 2.5: microcrack initiation criterion (based on the EPE solution) and the microcrack evolution function.
- Section 3 presents a study on the EPE based crack initiation criterion and discusses the characteristics of the rough microcrack surfaces. The sub-sections are arranged as follows;
  - Section 3.1: the interfacial transition zone (ITZ) of a two-phase composite.
  - Section 3.2: location of microcrack initiation.
  - Section 3.3: sensitivity of the EPE solution with respect to elastic properties.
  - Section 3.4: brief summary of findings from EPE study.
  - Section 3.5: variability of the crack surface roughness.
- Section 4 deals with the numerical implementation of the model and provides an algorithm for the stress calculation procedure.
- In Section 5, numerical predictions of uniaxial, biaxial and triaxial tests are compared with experimental results in order to evaluate the performance of the model. In addition a parametric study is presented on the effects of varying the roughness parameters.

2. Theoretical components of the model

The model simulates a two-phase composite comprising a matrix phase (m) that represents the mortar and spherical inclusions (Ω) that represent the coarse aggregate particles. The composite incorporates randomly distributed penny-shaped microcracks with rough surfaces on which stress can be recovered. It is assumed that the microcracks are initiated at the matrix-inclusion interface and then propagate through the matrix phase. This idealization is illustrated in Fig. 1.

2.1. Elastic two-phase composite

The average elastic properties of the two-phase composite and a relationship between the mean stress in the matrix phase and the average stress in the composite were determined by means of the classical Eshelby inclusion solution combined with the Mori–Tanaka homogenisation scheme for a non-dilute distribution of inclusions (Nemat-Nasser and Hori, 1999). Thus,

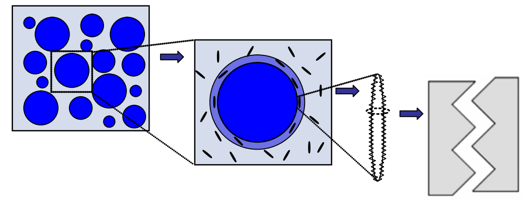

Fig. 1. Microcracking and rough contact concepts.

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_{m\Omega} : \bar{\boldsymbol{\varepsilon}}_e \tag{1}$$

$$\boldsymbol{\sigma}_m = \mathbf{W}_{m\Omega} : \bar{\boldsymbol{\sigma}} \tag{2}$$

where $\bar{\boldsymbol{\sigma}}$ and $\bar{\varepsilon}$ are the average or far-field stress and strain tensors respectively and where subscript e denotes elastic strain components. $\mathbf{D}_{m\Omega} = (f_\Omega \mathbf{D}_\Omega : \mathbf{T}_\Omega + f_m \mathbf{D}_m) : (f_\Omega \mathbf{T}_\Omega + f_m \mathbf{I}^{4s})^{-1}$, $\mathbf{W}_{m\Omega} = \mathbf{D}_m : (f_\Omega \mathbf{D}_\Omega : \mathbf{T}_\Omega + f_m \mathbf{D}_m)^{-1}$. f_β is the volume fraction of the β-phase ($\beta = m$ or Ω) and $f_m + f_\Omega = 1$. $\mathbf{T}_\Omega = (\mathbf{I}^{4s} + \mathbf{S}_\Omega : \mathbf{A}_\Omega)$, $\mathbf{A}_\Omega = [(\mathbf{D}_\Omega - \mathbf{D}_m) : \mathbf{S}_\Omega + \mathbf{D}_m]^{-1} : (\mathbf{D}_m - \mathbf{D}_\Omega)$, $\mathbf{S}_\Omega$ is the Eshelby tensor for spherical inclusions (Nemat-Nasser and Hori, 1999), $\mathbf{D}_\beta$ is the elasticity tensor of the β-phase and $\mathbf{I}^{4s}$ is the fourth order identity tensor.

2.2. Additional strain due to penny-shaped microcracks

The added strains from a dilute distribution of penny shaped cracks were next obtained from Budiansky and O'Connell (1976). For each set of microcracks with the same orientation, the added strain components are as follows:

$$\boldsymbol{\varepsilon}_\alpha = f\,\frac{16(1-\upsilon_m^2)}{3E_m}\begin{bmatrix} \sigma_{rr} \\ \frac{4}{2-\upsilon_m}\sigma_{rs} \\ \frac{4}{2-\upsilon_m}\sigma_{rt} \end{bmatrix} \tag{3}$$

where E_m is the Young's modulus of the matrix. f is the crack density parameter of Budiansky and O'Connell which was subsequently and more conveniently expressed in terms of a directional damage parameter ω that grows from zero (undamaged state) to one (fully damaged state). $\mathbf{r}$, $\mathbf{s}$, $\mathbf{t}$ define the unit local coordinate vectors, with $\mathbf{r}$ being the vector normal to the microcrack surface.

It proves convenient, particularly when rough contact (Section 2.4) is included in the formulation, to extract the added strains from an assumed band of matrix material which contains the microcracks, such that they are the same as those given by Eq. (3).

Defining the local elastic compliance tensor as $\mathbf{C}_L$, or in 'stiffness' form as $\mathbf{D}_L\left(=\mathbf{C}_L^{-1}\right)$, and noting that this has only non-zero components relating to rr, rs and rt components, as in Eq. (3), the local stress–strain relationship may be defined as follows

$$\mathbf{s} = (1-\omega)\mathbf{D}_L : \boldsymbol{\varepsilon}_L \tag{4}$$

in which $\mathbf{D}_L$, in reduced matrix form, is given by $\mathbf{D}_L = E_m\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2-\upsilon_m}{4} & 0 \\ 0 & 0 & \frac{2-\upsilon_m}{4} \end{bmatrix}$ and $\mathbf{s} = [\sigma_{rr}\ \sigma_{rs}\ \sigma_{rt}]^{\mathrm{T}}$ is the local stress tensor in reduced vector form.

From (3), the added microcracking component of the strain tensor, which is equal to $\boldsymbol{\varepsilon}_\alpha$, is given by

$$\boldsymbol{\varepsilon}_\alpha = \boldsymbol{\varepsilon}_L - \boldsymbol{\varepsilon}_{Le} = \left(\frac{1}{1-\omega} - 1\right)\mathbf{C}_L : \mathbf{s} \tag{5}$$

with $\boldsymbol{\varepsilon}_L$ being defined as the local strain within the microcracking material band.