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A micromechanics-based strain gradient damage model for fracture prediction of brittle materials – Part II: Damage modeling and numerical simulations

J. Li [a,*], T. Pham [a], R. Abdelmoula [a], F. Song [b], C.P. Jiang [c]

[a] LSPM, CNRS UPR 3407, Université Paris XIII, Villetaneuse, France
[b] State Key Laboratory of Nonlinear Mechanics, Institute of Mechanics, Chinese Academy of Sciences, Beijing, China
[c] Solid Mechanics Research Center, Beijing University of Aeronautics and Astronautics, Beijing, China

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ABSTRACT

In this paper, we established a strain-gradient damage model based on microcrack analysis for brittle materials. In order to construct a damage-evolution law including the strain-gradient effect, we proposed a resistance curve for microcrack growth before damage localization. By introducing this resistance curve into the strain-gradient constitutive law established in the first part of this work (Li, 2011), we obtained an energy potential that is capable to describe the evolution of damage during the loading. This damage model was furthermore implemented into a finite element code. By using this numerical tool, we carried out detailed numerical simulations on different specimens in order to assess the fracture process in brittle materials. The numerical results were compared with previous experimental results. From these studies, we can conclude that the strain gradient plays an important role in predicting fractures due to singular or non-singular stress concentrations and in assessing the size effect observed in experimental studies. Moreover, the self-regularization characteristic of the present damage model makes the numerical simulations insensitive to finite-element meshing. We believe that it can be utilized in fracture predictions for brittle or quasi-brittle materials in engineering applications.

1. Introduction

The fracture prediction of structures made of brittle materials is an important issue in engineering designs. In real structures, the failures are often initiated from a few geometrical weaknesses near which stress concentrations are formed. The stress concentrations are of many types and different levels. The failure prediction for all these stress concentrations is an essential research topic for scientists and engineers. However, it seems that fractures can be accurately predicted only for few types of stress states as so far. For brittle materials, failure criteria for two simple situations are commonly accepted:

1. Under uniform uniaxial tension, fracture occurs when the maximum tensile stress reaches the ultimate stress of the material:

$$\sigma \geqslant \sigma_c \tag{1.1}$$

2. For solids including a macrocrack, the crack grows when the Griffith criterion is fulfilled:

$$G \geqslant G_c \tag{1.2}$$

where G and G_c are respectively the energy release rate and its critical fracture value.

In the cases when the stress distribution is not uniform but does not present a crack singularity, these criteria are no longer sufficient to describe accurate fracture conditions. Many factors such like stress gradient, multi-axial stress state or structure size may influence the material strength (Bazant, 1976). With the aim of extending these criteria to more general cases, numerous fracture and damage models were proposed in the literature.

When the stress concentration presents a singularity weaker than the crack one, such a singularity can be found in the cases of sharp notches, interface cracking or cracks in ductile materials, criteria based on finite fracture mechanics were developed and reported in the literature. In simple words, these criteria are kinds of combinations of (1.1) and (1.2). (McClintock, 1958; Irwin, 1968; Ritchie et al., 1973; Seweryn and Lukaszewicz, 2002; Leguillon, 2002, etc.). Another class of fracture criteria was issued from the so-called cohesive models (Barenblatt, 1959; Dugdale, 1960). The cohesive models were successfully applied to fracture prediction of brittle or quasi-brittle materials (Xu and Needleman, 1994; Camacho and Ortiz, 1996; Foulk et al., 2000; Mohammed and Liechti, 2000, etc.). In all these criteria and models, one can distinguish a length scale parameter, such as the critical distance from the crack tip in finite fracture mechanics or the maximum

* Corresponding author. Address: LSPM, CNRS UPR 3407, Institut Galilée, Université Paris 13, 99 Avenue Jean-Baptiste Clément, 93430 Villetaneuse, France. Tel.: +33 1 49 40 28 89; fax: +33 1 49 40 39 38.
E-mail address: jia.li@univ-paris13.fr (J. Li).

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separation distance in cohesive models. The introduction of a length parameter allows for a more correct description of fracture and size effect. In general, these length scale parameters are not independent and can be related to σ_c and G_c. Li and Zhang (2006) have adapted a large number of these criteria to predict fractures caused by a non-singular stress concentration. It was shown that all of them predict too conservative critical loads compared to the experimental results.

The finite fracture concept was also adapted in damage analyses. A wide variety of damage models have been proposed on the basis of continuum damage mechanics (Lemaitre and Chaboche, 1990) by introducing a length parameter. Pijaudier-Cabot and Bazant (1987) developed a practical nonlocal model in a continuum damage setting. The development of nonlinear gradient models has taken place predominately in plasticity (Aifantis, 1984; Lasry and Belytschko, 1988; de Borst and Muhlhaus, 1992; Nix and Gao, 1998; Gao et al., 1999; Fleck and Hutchinson, 1993, etc). These gradient theories have afterwards been transferred to damage mechanics (Peerlings et al., 1996; Frémond and Nedjar, 1996, etc.). The cohesive concept was extended to a so-called "Virtual Internal Bond" model by Gao and Klein (1998), that transforms the surface cohesive model into a volume cohesive model (Nguyen et al., 2004; Thiagarajan et al., 2004; Zhang and Ge, 2006, etc.). Francfort and Marigo (1998) developed a so-called damage gradient model in which the damage zone converges to a crack as the length parameter tends to zero (Bourdin et al., 2000). Li (2008) developed a cohesive model with strain gradient effect.

Introduction of length scale parameters into fracture mechanics or damage models present several evident advantages. First, fractures can be predicted for some cases of stress concentrations different from that near a crack tip. Second, size effect can be dealt with, especially for the cases when the scale of observation approaches that of the microstructure of the material. Third, mesh-dependent phenomenon in numerical simulations can be attenuated by the regularization effect of this approach. However, it has to be noted that in most of these models, the length parameters were only adopted for numerical convenience. Their physical meanings in constitutive equations and their microscopic origins incite always debates and discussions.

In the first part of this work (Li, 2011), we have established a strain-gradient constitutive law for linear-elastic materials containing microcracks by using a special homogenization procedure. In this constitutive law, the length parameter that modulates the strain gradient effect is defined with a clear physical meaning: it is directly related to the average length of the microcracks. Even though the obtained strain energy density is in similar form to those proposed in the literature, it benefits from its physical clarity with all the parameters measurable by means of experimentations.

In the present work, this strain-gradient constitutive law is transformed to a damage model. To this end, we first proposed a resistance curve for microcrack growth before damage localization. By introducing this resistance curve into the strain gradient constitutive law established in the first part of this work, we obtained an energy potential that is capable to provide a complete stress–strain relationship with evolution of damage. Then this damage model was implemented into a finite element code. Using this numerical tool, we carried out detailed numerical simulations on different specimens in order to study the influence of strain gradient. Comparisons with previous experimental results show that strain gradient plays an important role in predicting fracture due to singular or non-singular stress concentrations. The size effect of fracture observed in experimental studies can also be assessed. Moreover, the present damage model presents a regularization effect in numerical simulations. Even though the present strain gradient damage model was established on the basis of simple physical concepts, the obtained results are quite encouraging.

The outline of the paper is as follows. In Section 2, we establish the strain-gradient damage evolution law. In Section 3, we describe the implementation of the model into a finite element code; In Section 4, detailed numerical simulations on different specimens made of brittle materials are presented and commented. Comparisons with previous experimental results were carried out in order to validate the present model. Finally, in Section 5, we present some concluding remarks in relation to the present strain-gradient damage model.

2. Damage modeling

In this section, we will consider the microcrack propagation within a Representative Volume Element (RVE) in order to establish an evolution law of damage including the strain-gradient effect. The starting point is the strain energy function (4.22) established in the first part of this work (Li, 2011), namely:

$$U = \frac{1}{2}\left[(1-\pi\rho)C^0_{ijpq}\varepsilon_{ij}\varepsilon_{pq} + \frac{\pi\bar{a}^2}{12}C^0_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right] \tag{2.1}$$

where U stands for the strain energy density of a two-dimensional body containing many microcracks; ρ is the microcrack density; $\bar{a} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(a^{(n)})^2}$ is the mean square root of the semi-crack lengths, by denoting V^0 the average volume containing one microcrack, we have $\rho = \sum_{n=1}^{N}\frac{a^{(n)2}}{V} = \frac{\bar{a}^2}{V^0}$; $\mathbf{C}^0$ is the elastic stiffness tensor of the virgin material without microcracks and ε is the strain tensor.

From this expression, we can remark that when the microcracks grow, i.e., when $\bar{a}$ increases, on the one hand, the stiffness associated with strain decreases since the crack density ρ increases; on the other hand, the stiffness associated with strain gradient increases. Consequently, strain gradient will influence the constitutive behavior as well as the evolution of damage.

Now let us consider the variation of the strain energy density with respect to the increment of strain, strain gradient and average microcrack length:

$$\Delta U = (1-\pi\rho)C^0_{ijpq}\Delta\varepsilon_{ij}\varepsilon_{pq} + \frac{\pi\bar{a}^2}{12}C^0_{ijpq}\delta_{kr}\Delta\varepsilon_{ij,k}\varepsilon_{pq,r} - \left(C^0_{ijpq}\varepsilon_{ij}\varepsilon_{pq} - \frac{V^0}{12}C^0_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right)\frac{\pi\bar{a}}{V^0}\Delta\bar{a} \tag{2.2}$$

Since the increment of the average microcrack length is irreversible, the third term in the right-hand side of (2.2) corresponds to the energy dissipation increment due to the crack growth. The generalized thermodynamic force associated with $\Delta\bar{a}$ is, according to Lemaitre and Chaboche (1990):

$$G = -\frac{\partial U}{\partial \bar{a}} = \left(C^0_{ijpq}\varepsilon_{ij}\varepsilon_{pq} - \frac{V^0}{12}C^0_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right)\frac{\pi\bar{a}}{V^0} \tag{2.3}$$

G represents here the dissipative energy ratio due a to unit increment of $\bar{a}$ in a unit volume. It can be considered as the energy release rate in a more general sense. According to the Griffith criterion, there is crack propagation when G attains its critical value $G_c(\bar{a})$. Therefore the following inequality must hold:

$$G \leqslant G_c(\bar{a}) \tag{2.4}$$

Here we assume that the critical energy release rate varies as function of $\bar{a}$. This assumption is supported by numerous experimental observations (Frost, 1959; Kitagawa and Takahashi, 1976; Lankford, 1985 among others). These authors reported that small cracks grow more quickly than large cracks in fatigue tests under the same loading level measured by energy release rate. As a consequence,