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Periodic crack problem for a functionally graded half-plane an analytic solution

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ABSTRACT

The plane elasticity problem of a functionally graded semi-infinite plane, containing periodic imbedded or edge cracks perpendicular to the free surface is considered. Cracks are subjected to mode one mechanical or thermal loadings, which are represented by crack surface tractions. Young's modulus, conduction coefficient, coefficient of thermal expansion are taken as exponentially varying functions of the depth coordinate where as Poisson ratio and thermal diffusivity are assumed to be constant. Fourier integrals and Fourier series are used in the formulation which lead to a Cauchy type singular integral equation. The unknown function which is the derivative of crack surface displacement is numerically solved and used in the calculation of stress intensity factors. Limited finite element calculations are done for verification of the results which demonstrate the strong dependence of stress intensity factors on geometric and material parameters.

1. Introduction

Functionally graded materials (FGMs) are nontraditional engineering materials that are used especially in coating applications such as thermal barrier or wear resistant coatings (Erdogan, 1995). They are inhomogeneous materials whose properties vary in a specified manner. In order to gain a better understanding of the fracture of functionally graded materials, different researchers have solved many crack problems associated with FGMs within the last couple of decades. Employing various material modeling approaches and solution techniques, many useful results have been obtained for different crack configurations and loading conditions. There exist now, a vast literature on this subject. A relatively less studied subject within this vast literature pertains to the periodic cracking of functionally graded materials.

In this study one such problem, namely, plane strain mode one periodic cracking of a functionally graded semi-infinite plane is considered. Granted, the problem at hand is a highly idealized one and even more realistic problems (such as an FGM coating bonded to a half plane) have already been solved as will be discussed in the forthcoming literature survey. The distinguishing feature of this study, however, is that an analytical solution with a certain subtlety (to be discussed in Section 2) is being presented. In the earlier studies, either finite element or some other approximate method is used; or analytical solutions are obtained for some special materials which have variable thermal properties but constant elastic properties. Hence to the best of authors' knowledge, the analytical solution to the problem presented here, has not been published yet. The relevant literature is briefly reviewed in the following. The scope of the literature survey is restricted to periodic crack problems in linear elastic materials under thermal or mechanical loads.

Periodic cracks in homogeneous materials are investigated by many researchers. Earliest works belong to Benthem and Koiter (1973), and Bowie (1973) who solved the problem of a half-plane with periodic edge cracks by using different approaches. Nisitani et al. (1973) considered a row of internal cracks in a semi-infinite plane under uniform tension. Nemat-Nasser et al. (1978) addressed the issue of stability of crack growth under specific thermal stress conditions by considering a half plane containing two sets of interacting periodic edge cracks which are equally spaced but of unequal lengths. Stress intensity factors (SIF) used in the stability analysis are obtained from the solution of a singular integral equation with Cauchy type singularity. Isida (1979) also considered an array of parallel edge cracks in a semi-infinite plane under uniform tension. Some results from Nisitani et al. (1973) and Isida (1979) are given in Murakami (1987). Nied (1987) considered the elasticity problem of an infinite array of periodic internal cracks in a half plane, such that edge cracks could also be obtained as a special case. Dependence of SIFs and crack opening displacements on crack spacing has been investigated. In Nied (1987), instead of taking the usual approach of formulating the problem by using the derivative of the crack surface displacement (which leads to a singular integral equation

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of the Cauchy type), an alternative approach has been followed in which the unknown function is taken to be the crack surface displacement, and this approach results in a hypersingular integral equation. The problem was formulated first, for a single crack by using Fourier transforms and then the expressions for multiple cracks were obtained by using superposition. Later, Schulze and Erdogan (1998) considered the periodic cracking of a homogeneous elastic coating bonded to a homogeneous substrate. They expressed the displacement field in the cracked medium as the superposition of Fourier integrals and Fourier series and formulated the problem in terms of the crack surface displacement, which led to a hypersingular integral equation.

Thermal stress problems for homogeneous media containing periodic cracks have been considered by Rizk (2003, 2005, 2006) in a series of articles. In Rizk (2003) periodic cracks in a half-plane which is subjected to convective cooling is considered. The thermal stress problem for the crack free medium was solved first and then the thermal stresses with the opposite sign were applied as the crack surface tractions. The problem was formulated for a single crack by using Fourier transforms and the derivative of crack surface displacement as the unknown function. Superposition was used to obtain the required expressions for multiple cracks as in Nied (1987) and a singular integral equation with Cauchy type singularity was obtained. In Rizk (2005) and Rizk (2006) different periodic crack configurations in elastic strips have been considered. In these studies, the problems were formulated by using Fourier series and Fourier Integrals in terms of the crack surface displacements, which led to hypersingular integral equations. In all of these studies transient SIFs have been presented. Multiple cracking of an elastic homogeneous coating under transient thermal load has been addressed by Wang and Mai (2007). In Schulze and Erdogan (1998) and Wang and Mai (2007), the overall geometry of the cracked body and the solution methods were the same, the only difference being the applied loading. A more recent contribution in this area came from Jin and Feng (2009), where a homogeneous elastic strip containing an array of parallel edge cracks with alternating lengths subjected to a thermal shock, is considered. A Fourier transform-superposition method, similar to that in Rizk (2003) was used, which led to a system of singular integral equations (Cauchy type) for the unknown crack surface displacement derivatives. Thermal SIFs were presented.

In all of the studies reviewed so far plane strain or plane stress solutions were obtained.

Since mid 1990's, solutions to periodic crack problems in functionally graded materials have started to appear in the technical literature. Two main venues followed by the researchers to obtain solutions are the integral equation method and the finite element method (FEM). Here, the analytical solutions are reviewed first in some detail. Finite element studies will be briefly mentioned next.

Periodic cracking of a functionally graded coating bonded to a homogeneous substrate under anti-plane loading has been considered by Erdogan and Ozturk (1995). Functional grading was represented as an exponentially varying modulus of rigidity. A hypersingular integral equation whose unknown is the crack surface displacement, was derived and mode III SIFs, stresses, crack opening displacements and strain energy released per unit surface area were calculated. Later Chen (2006) and, Wang and Mai (2006a) also considered anti-plane problem of periodic cracks in graded coatings but for transient (dynamic) loading. In both of these articles, time dependence had been taken care of by using Laplace transforms, and hypersingular integral equations are obtained by using Fourier transforms and Fourier series for unknown crack surface displacements. Recently, anti-plane problem of periodic interface cracks in a functionally graded coating-substrate structure has been considered by Ding and Li (2008).

Choi (1997) considered periodic imbedded cracks in an infinite non-homogeneous medium loaded under in-plane normal and shear stresses. The non-homogeneity is represented as an exponentially varying modulus of rigidity in the direction of the cracks, whereas Poisson ratio is taken as constant. Hypersingular integral equations (whose unknowns are the crack surface displacements) for each individual loading mode are derived and solved. SIFs and crack surface displacements have been presented. Later, Wang and Mai considered the same kind of material parameter variation and crack configuration for the cases of thermo-mechanical (Wang and Mai, 2005) and transient (dynamic) loading (Wang and Mai, 2006b). In an earlier study, Afsar and Sekine (2000) discussed the effect of crack spacing on the brittle fracture characteristics of a semi-infinite functionally graded material with periodic edge cracks. Their approach to material modeling, however, has been quite different. Quoting, "The non-homogeneity of the material is simulated by an equivalent eigenstrain, whereby the problem is reduced to that of a cracked homogeneous material with incompatible and equivalent eigenstrains." Recently, Jin and Feng considered multiple edge crack problems under thermal shock for a thermally graded but elastically homogeneous plate (Jin and Feng, 2008a) and for a thermally graded but elastically homogeneous coating on a homogeneous substrate (Jin and Feng, 2008b). Feng and Jin (2009) also considered thermal fracture of a thermally graded but elastically homogeneous plate containing two sets of interacting periodic edge cracks which are equally spaced but of unequal lengths. In all of these studies, a Fourier transform-superposition method similar to those in Nied (1987) and (Rizk, 2003) are used and, the unknown being the crack surface displacement derivative, singular integral equations with Cauchy type singularity are obtained. Thermal SIFs were calculated and predictions regarding thermal shock resistance were made.

Periodic crack problems in functionally graded materials have also been addressed by using FEM. Thermal and mechanical loading of coating-substrate systems (Bao and Wang, 1995), thermal loading with temperature dependent material parameters (Ueda, 2002), thermal shock enhancement due to multiple cracking (Han and Wang, 2006) and three dimensional crack problems (Dag et al., 2008) have been considered.

The current study presents the analytic solution of plane strain mode one periodic crack problem of a functionally graded semi-infinite plane. Spatial variation of the Young's modulus is taken as an exponential function, and Poisson ratio is taken as constant. The problem has been reduced to a perturbation problem in which the crack surface tractions are the only non-zero external loads. Following (Schulze and Erdogan, 1998), a Fourier integral-Fourier series representation of displacements is used in the formulation, but a Cauchy type singular integral equation is derived since the auxiliary unknown function is selected as the derivative of the crack surface displacement rather than the displacement. In this respect, the given solution is unique considering the literature survey given above. The analytic approach developed here can be extended to more realistic cases such as functionally graded strips or coatings containing periodic cracks. The main objective of this study is to examine the effect of length parameters (i.e. crack location, spacing and crack length) and material grading on SIFs for imbedded and edge cracks. SIFS are given for general loading conditions as well as constant strain and thermal shock loadings.

2. Formulation of the problem

The geometry of the plane elasticity problem of periodic cracks in a functionally graded semi-infinite medium is given in Fig. 1. By