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A coupled discrete/continuous method for computing lattices. Application to a masonry-like structure

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ABSTRACT

This paper presents a coupled discrete/continuous method for computing lattices and its application to a masonry-like structure. This method was proposed and validated in the case of a one dimensional (1D) railway track example presented in Hammoud et al. (2010). We study here a 2D model which consists of a regular lattice of square rigid grains interacting by their elastic interfaces in order to prove the feasibility and the robustness of our coupled method and highlight its advantages. Two models have been developed, a discrete one and a continuous one. In the discrete model, the grains which form the lattice are considered as rigid bodies connected by elastic interfaces (elastic thin joints). In other words, the lattice is seen as a "skeleton" in which the interactions between the rigid grains are represented by forces and moments which depend on their relative displacements and rotations. The continuous model is based on the homogenization of the discrete model (Cecchi and Sab, 2009). Considering the case of singularities within the lattice (a crack for example), we develop a coupled model which uses the discrete model in singular zones (zones where the discrete model cannot be homogenized), and the continuous model elsewhere. A new criterion of coupling is developed and applied at the interface between the discrete and the continuum zones. It verifies the convergence of the coupled solution to the discrete one and limits the size of the discrete zone. A good agreement between the full discrete model and the coupled one is obtained. By using the coupled model, an important reduction in the number of degrees of freedom and in the computation time compared to that needed for the discrete approach, is observed.

1. Introduction

The aim of this paper is to propose an extension to 2D structures of the 1D coupled method between discrete and continuum media proposed in Hammoud et al. (2010). We focus here on the robustness and the feasibility of the coupled method in the presence of cracks and stress concentrations.

Actually, the 1D model studied in Hammoud et al. (2010) consisted of a beam resting on an elastic springs. The deflection of the beam (as well as the nodal parameters) was calculated by using two approaches; a discrete approach and a macroscopic approach deduced from the discrete one. A comparison between the response of the system obtained by using these approaches showed the cases where the macroscopic approach cannot replace the discrete one. This difference leaded us to apply a discrete/continuum coupling method. A new criterion of coupling was developed and applied at the interface of the discrete and continuum subdomains. In the coupled approach, the macroscopic scale was the intial scale computation. A local discrete computation was done on each macroscopic element. A comparison was done between the nodal parameters computed by the local discrete method and the continuum one. If a strong difference was observed, a refinement of the computation scale was done. This procedure of refinement was necessary in the zone of singularities.

In this present research, a 2D model will be considered. A masonry pannel can be described by a discrete model or a continuous model. See Alpa and Monetto (1994), Sab (1996), Cecchi and Sab (2004), Cecchi and Sab (2004) and Cluni and Gusella (2004), for example. In the discrete model, the blocks which form the masonry wall are modeled as rigid bodies connected by elastic interfaces. Then, the masonry is seen as "skeleton" in which the interactions between the rigid blocks are represented by forces and moments which depend on their relative displacements and rotations. The second model is a continuous one based on the homogenization of the discrete model. The aim of this paper is to extend the 1D coupled method of Hammoud et al. (2010) to 2D structures in the presence of cracks and stress concentrations.

Many coupled approaches between discrete and continuum media were developed. See among others the works of Broughton et al. (1999), Curtin and Miller (2003), Wagner and Liu (2003),

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Fish and Chen (2004), Xiao and Belytschko (2004), Ricci et al. (2005), Klein and Zimmerman (2006), Rousseau et al. (2008) and Rousseau et al. (2009). In these works, the domain is decomposed into sub-domains; discrete, continuum and an interface between the discrete and continuum sub-domains. A handshake zone where the two descriptions of material figure can exist at the interface sub-domain. These approaches are divided into energy-based or force-based formulations. Briefly, in the energy-based formulation, it is assumed that the total energy of a domain can be written as the sum of the energy of the three sub-domains discrete, continuum and handshake from which it is composed. The energy of the handshake region is a partition-of-unity blending of discrete and continuum energy descriptions. For example, a well-known energy-based method which includes a handshake region is the bridging domain (BD) method described in Xiao and Belytschko (2004). Within the BD handshake region, both the continuum and discrete energies are used, but their contributions are weighted according to a function θ that varies linearly from 1 at the edge of the handshake region closest to the continuum one to 0 at the edge closest to the discrete zone. The total energy is then minimized subject to the imposed displacement boundary conditions to obtain the equilibrium configuration of the system. The approximation inherent to an energy-based coupling of this type leads to errors known as "**ghost forces**". The ghost forces are defined as follows: Consider a model in which the discrete elements are on their equilibrium state, and the finite elements are unstressed and undeformed. Physically, this should be an equilibrium configuration where all forces are zero, and therefore any residual forces on the discrete element or nodes that arise in this configuration are unphysical and will lead to spurious distortions of the domain upon relaxation. These unphysical forces are the ghost forces. The existence of ghost forces is, it seems, a necessary consequence of having well-defined energy functional. All energy-based methods mentioned above, suffer from these forces in various degrees. An alternate approach is to abandon the energy-based approach and instead starts from the forces directly. Methods of this type can indeed eliminate the ghost forces (see Kohlhoff et al., 1991). In these approaches, there is no handshake zone and strong compatibility sets the position of the discrete element and the nodes along the interface zone. A force-based method is based on the following philosophy: to eliminate ghost forces, design the method so that the forces are identically zero when the perfect discrete sub-domain is in its correct equilibrium state. Since it does not seem possible to do this in general using an energy functional, we derive forces without recourse to a total energy. For more details, an exhaustive literature review of these coupled models has been given in Hammoud et al. (2010).

In the energy-based and the force-based formulations, the size of the discrete zone is not defined. It depends on many parameters as the weight function, the boundary conditions at the interface zone, etc. In our force-based formulation, there is no handshake zone and the discrete zone that contains the singularities is fixed at the beginning of the simulation. It will be controlled by a special coupling criterion at the interface zone, described in (Section 4.2.2). If the value of this criterion is not small enough to ensure a convergence, the size of the discrete zone will be increased. This iterative procedure is repeated until convergence of the coupled solution to the discrete one is reached.

As for the 1D model (Hammoud et al., 2010), the mechanical parameters of the system being studied will be calculated in a way that does not require the calculation of the energy and avoids the problem of how to partition this energy between the discrete and continuum zones at the interface. We will calculate the global rigidity matrices (discrete ($\mathbf{K}^D$), continuous ($\mathbf{K}^C$) and interaction ($\mathbf{K}^{C-D}$)) and then solve a linear system written as follows:

$$\underbrace{\left[\left(\begin{array}{c|c}\mathbf{K}^C & 0\\ \hline 0 & \mathbf{K}^D\end{array}\right)+(\mathbf{K}^{C\text{-}D})\right]}_{\mathbf{K}^{total}}\begin{pmatrix}U^C\\ U^D\end{pmatrix}=\mathbf{F}^{total}. \tag{1}$$

In this present research, at first, we present the 2D masonry model. Secondly, we develop the discrete and the continuous models used to calculate the behavior of the masonry pannel. The continuous boundary value problem is solved by using the Finite Element Method. We implement the full continuous and the full discrete models in a MATLAB code as well as the coupled discrete/continuous one. This case is validated in comparison with a FE software (ABAQUS). We also develop a numerical bench test in order to prove that the discrete medium is homogenizable in the case of no singularities. In the case where singularities exist in the structure (a crack for example), a criterion of coupling between discrete and continuous models, is developed. Near the crack, a discrete zone is used and farther a FE mesh is employed. The criterion of coupling applied at the interface of these zones, verify the convergence of the coupled solution to that discrete. The size of the discrete zone is limited and a considerable reduction of the DoFs is also observed.

2. The discrete model

The 2D model consists of a regular lattice of square rigid grains interacting by their elastic interfaces (see Fig. 1).

The in-plane motion of the grain can is described by two displacements and one rotation at the center.

The geometry of the lattice is described hereafter. The position of the center of grain $B^{i,j}$, $\mathbf{y}^{i,j}$, in the Euclidean space is formulated as follows:

$$\mathbf{y}^{i,j}=ia\mathbf{e_1}+ja\mathbf{e_2}, \tag{2}$$

$\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{e}_{bf3}$ is an orthonormal base.

So the displacement of the $B^{i,j}$ grain is an in plane rigid body motion:

$$\mathbf{u}(\mathbf{y})=\mathbf{u}^{i,j}+\boldsymbol{\omega}^{i,j}\times(\mathbf{y}-\mathbf{y^{i,j}}),\quad\forall\mathbf{y}\in B^{i,j} \tag{3}$$

where

$$\mathbf{u}^{i,j}=u_1^{i,j}\mathbf{e_1}+u_2^{i,j}\mathbf{e_2}\quad\text{and}\quad\boldsymbol{\omega}^{i,j}=\omega_3^{i,j}\mathbf{e_3}. \tag{4}$$

If the mortar joint is modeled as an elastic interface, then the constitutive law is a linear relation between the tractions on the block surfaces and the jump of the displacement:

$$\mathbf{t}=\boldsymbol{\sigma}\ \mathbf{n}=\mathbf{K}\cdot\mathbf{d}\quad\text{on }S. \tag{5}$$

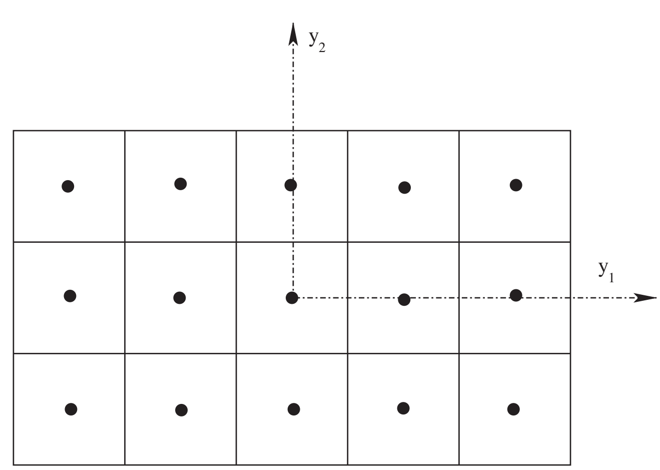

Fig. 1. Square grains forming the regular lattice.