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Elastic interaction of partially debonded circular inclusions. II. Application to fibrous composite

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ABSTRACT

A complete analytical solution has been obtained of the elasticity problem for a plane containing periodically distributed, partially debonded circular inclusions, regarded as the representative unit cell model of fibrous composite with interface damage. The displacement solution is written in terms of periodic complex potentials and extends the approach recently developed by Kushch et al. (2010) to the cell type models. By analytical averaging the local strain and stress fields, the exact formulas for the effective transverse elastic moduli have been derived. A series of the test problems have been solved to check an accuracy and numerical efficiency of the method. An effect of interface crack density on the effective elastic moduli of periodic and random structure FRC with interface damage has been evaluated. The developed approach provides a detailed analysis of the progressive debonding phenomenon including the interface cracks cluster formation, overall stiffness reduction and damage-induced anisotropy of the effective elastic moduli of composite.

1. Introduction

Mechanical behavior of composite materials is significantly affected by the degree of bonding between the constituents. Interfacial debonding in the form of arc crack is the predominant damage mode of unidirectional fiber reinforced composite (FRC) under transverse loading, which results in rapid loss of stiffness and strength. From the viewpoint of damage-tolerant design of composite structures, it is important to find an optimal balance between the strength and stiffness. To achieve this goal, an effect of interface debonding on the performance of composite needs to be fully understood and adequately implemented in the predictive models.

There is a body of publications on stiffness prediction for fibrous composites with interfacial debonding by means of micromechanics. However, the most existing work is based on models of varying degrees of approximation in the treatment of fiber interaction and local stress and strain fields. Among them, we mention the "dilute" model (Ju, 1991; Lee and Simunovic, 2001) where interaction of fibers is neglected. In the "composite cylinder" model used by Teng (1992) and Tandon and Pagano (1996), interaction between the fibers is taken into account in the self-consistent manner. The Mori–Tanaka method-based model by Takahashi and Chou (1988) assumes debonding to take place over the entire interface. A dual effective-medium and finite-element study was carried out by Zheng et al. (2000) and Zhao and Weng (2002) for a single partially debonded elliptic fiber. In the effective-medium approach, the fictitious perfectly bonded inclusions were used and the Mori–Tanaka theory was applied to compute the effective moduli of composite. To our knowledge, only a few models for the multiple fibers with imperfect interface are available in literature. Wriggers et al. (1998) considered debonding by a contact formulation which can handle adhesional forces up to a prescribed tensile limit on the contact interface. In the work by Ghosh et al. (2000), interfacial debonding is accommodated by cohesive zone model, with the normal and tangential springs tractions expressed in terms of interfacial separation.

In the unit cell approach (Nemat-Nasser et al., 1982; Golovchan et al., 1993; Chen and Papathanasiou, 2004; Kushch et al., 2008; among others), an actual FRC is modeled by the equivalent periodic microstructure with a unit cell containing a certain number of inclusions. This model is advantageous in that it takes into account interactions of a whole infinite array of inhomogeneities whereas its deterministic geometry enables an accurate solution of the model problem. In application to the above problem, only the simplest models of this kind, namely, a composite containing a square or hexagonal array of equally debonded fibers, were considered. In the elastic contact model by Shan and Chou (1995), the fiber–matrix interface is assumed completely debonded. Yuan et al. (1997) simulated the debonded interface by uni- and doubly-symmetric interface cracks. In these papers, the interface bonding conditions are fixed in the problem statement. The interfacial failure option

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was implemented by Yeh (1992) and Aghdam et al. (2008) by means of interface/interphase elements and by Caporale et al. (2006) who modeled interfacial failure by the brittle-elastic springs. The model by Teng (2007) contains a periodic square array of fibers, with a random subset of them assigned to be completely debonded.

In the first part of this work (Kushch et al., 2010), a complete analytical solution has been derived for a finite array of interacting, partially debonded circular inclusions. The method combines the superposition principle with the complex potentials technique and results in a simple and efficient numerical algorithm. Here, we extend this approach to the cell model of fibrous composite with interface cracks. The multiple fiber cell geometry provides seamless joint with the adjacent cells. A complete analytical solution in displacements has been obtained in terms of the periodic complex potentials. By analytical averaging the local strain and stress fields, the exact expressions of the effective transverse elastic moduli have been found. Several test problems have been solved to check an accuracy and numerical efficiency of the method. An effect of interface crack density and clustering on macroscopic stiffness of FRC is estimated.

2. The model problem statement

2.1. Model geometry

Following Kushch et al. (2008), we consider a quasi-random model of unbounded fibrous composite solid. The model geometry is periodic, with a period a along the axes Ox_1 and Ox_2. For this geometry, any arbitrarily placed, oriented along the Ox_1 and Ox_2 axes square with side length a can be taken as its representative unit cell (RUC). It contains the centres of N_p of aligned in x_3-direction and circular in cross-section fibers (Fig. 1): within a cell, the fibers are placed randomly but without overlapping. The fibers shown by the dashed line do not belong to the cell while occupying a certain area within it. The whole composite bulk is obtained by translating the cell in two orthogonal directions.

It should be clearly stated that we formulate and solve the model problem for a whole composite plane rather than for the RUC. The RUC concept, however, is convenient for introducing the model geometry and averaging the periodic strain and stress fields in Section 4 and we use it for this purpose. We define geometry of the unit cell by its side length a and the coordinates (X_{1q}, X_{2q}) of qth fiber center $(q = 1, 2, \ldots, N_p)$ in the global Cartesian coordinate system Ox_1x_2. Number N_p can be taken large sufficiently to simulate micro structure of an actual disordered composite. We assume the fibers equally sized, of radius $R = 1$, and made from the same material. The fiber volume content is $c = N_p\pi/a^2$.

We introduce also the local, fiber-related coordinate systems $Ox_{1q}x_{2q}$ with origins in Z_q and will use the complex-value variables

$$z = x_1 + \mathrm{i}x_2, \quad z_q = x_{1q} + \mathrm{i}x_{2q}; \tag{1}$$

representing the point $x = (x_1, x_2)^T$ in the complex planes Ox_1x_2 and $Ox_{1q}x_{2q}$, respectively. The global complex variable $z = z_q + Z_q$, where $Z_q = X_{1q} + \mathrm{i}X_{2q},\ q = 1, 2, \ldots, N_p$. The local variables $z_p = z - Z_p$ relate each other by $z_q = z_p - Z_{pq}$. Here, $Z_{pq} = X_{1pq} + \mathrm{i}X_{2pq}$ is the complex number determining relative position of the fibers with indices p and q *inside* the cell, see Fig. 1. Also, we introduce the complex number Z^*_{pq} defining the minimal distance between the *arrays* of fibers with indices p and q. Here, we do not assume that the both fibers are belonging to the same RUC, see Fig. 1. The introduced numbers are related by $\left|Z^*_{pq}\right| = \min_{-1\leqslant k_1,k_2\leqslant 1}|Z_{pq} + (k_1 + \mathrm{i}k_2)a|$.

In these notations, the non-overlapping condition of any two fibers in entire composite space is written as $\left|Z^*_{pq}\right| > 2R$. In order to alleviate an analysis of the model problem, the small positive parameter known as the "minimum allowable inter-fiber spacing" $\delta_{\min} = \min_{p,q}\left|Z^*_{pq}\right|/2R - 1$ is often introduced in the multiple fiber models of FRC (e.g., Chen and Papathanasiou, 2004). In our numerical study, we put $\delta_{\min} = 0.01$. The geometry shown in Fig. 1 is generated using the molecular dynamics algorithm of growing particles (e.g., Kushch et al., 2008). This model was studied by several investigators under assumption that the matrix and fibers are perfectly bonded along the interfaces $L_q,\ q = 1, 2, \ldots, N_p$. We consider more general case by assuming the part $L_c^{(q)}$ of interface L_q defined by the endpoints $z_j^{(q)} = \exp(\mathrm{i}\theta_j^{(q)})\ (j = 1, 2)$ (Fig. 2) separated and the part $L_b^{(q)} = L_q \setminus L_c^{(q)}$ being perfectly bonded. In order to simplify the subsequent formulas, we introduce the crack-related complex variables $\zeta_q = z_q/z_c^{(q)}$, where $z_c^{(q)} = \exp(i\theta_c^{(q)})$ is the crack midpoint: $\theta_c^{(q)} = (\theta_1^{(q)} + \theta_2^{(q)})/2$. The interface crack size is measured by the angle $\theta_2^{(q)} - \theta_1^{(q)} = 2\theta_d^{(q)}$; in the prefect bonding case, $\theta_d^{(q)} = 0$. To the best knowledge of the authors, the multiple fiber RUC model with interface cracks never been considered before.

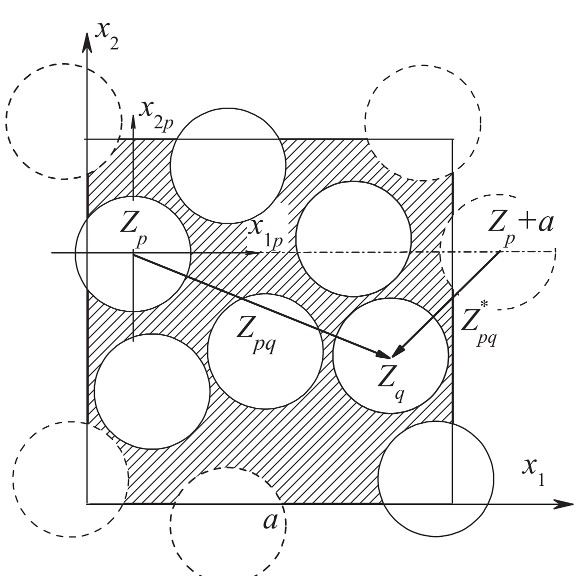

Fig. 1. RUC model of fibrous composite.

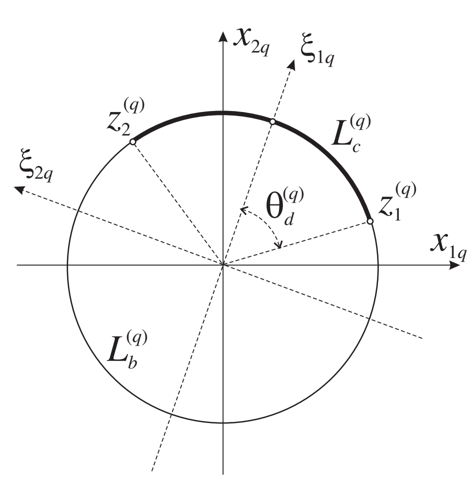

Fig. 2. A fiber with interface crack.