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Micro-cracks informed damage models for brittle solids

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ABSTRACT

A class of micro-cracks informed damage models for describing the softening behavior of brittle solids is proposed, in which damage evolution is treated as a consequence of micro-crack propagation. The homogenized stress–strain relation in the cracked microscopic cell defines the degradation tensor, which can be obtained by the equivalence between the averaged strain energy of the microscopic cell and the strain energy density of the homogenized material. This energy equivalence relationship serves as an energy bridging vehicle between the damaged continuum and the cracked microstructure. Several damage evolution equations are obtained by this energy bridging method. The size effect of the micro-cracks informed damage law is characterized through the microscopic cell analysis, and the proper scaling of the characterized damage evolution functions to eliminate mesh dependency in the continuum solution is introduced.

1. Introduction

Continuum damage mechanics (CDM) has been an active research area for more than 50 years (Kachanov, 1958; Robotnov, 1968; Lemaitre and Chaboche, 1974; Mazars, 1984; Ju, 1989; Faria et al., 1998). However, CDM is generally phenomenological, where the damage evolution functions are not related to specific microstructural parameters and local defects. Some earlier efforts have been devoted to the micromechanical investigation of cracked solids (Budiansky and O'connell, 1976; Norris, 1985; Hashin, 1988), however the link to CDM is lacking.

Damage evolution based on a multi-scale framework has been investigated in recent years. Lee et al. (1999) combined the standard finite element method (FEM) with asymptotic homogenization in the undamaged domain and the voronoi cell finite element model (VCFEM) in the damaged region. Fish et al. (1999) proposed a nonlocal damage theory based on asymptotic homogenization to account for damage effects in a heterogeneous media. The reliability of this approach, however, depends on reliability of the micro-scale damage models. Dascalu et al. (2008) and Dascalu (2009) defined the damage variable directly by the micro-crack length. Then a series of differential equations was introduced to describe damage evolution in the microscopic cell based on the homogenization theory and the energy controlled crack propagation. By introducing an internal length in the damage evolution equation, the size effect was characterized.

Damage processes in brittle solids are driven by the distribution of micro-cracks and their evolution. Explicit modeling of micro-cracks in brittle solids is computationally infeasible, while classical damage models are phenomenological in nature; missing the link to microscopic properties. Micromechanics based approaches have been introduced (Nemat-Nasser and Hori, 1999) to obtain homogenized material properties of cracked solids. However, they are limited to certain idealized crack configurations and loading conditions. Alternatively, asymptotic expansion offers a rigorous means for relating physical variables defined at different scales (Bensoussan et al., 1978; Bakhvalov and Panasenko, 1989; Guedes and Kikuchi, 1990; Fish et al., 1997). However, the key step in an asymptotic type method is solution of the characteristic functions in the microscopic cell, which is typically very time consuming.

The present work aims at constructing damage models based on thermodynamics of "cracked" microscopic cells and the corresponding "damaged" macroscopic continua. Crack evolution in the microscopic cell is first modeled numerically. Then, corresponding damage evolution functions in the continuum are constructed through a Helmholtz free energy relationship between damaged and undamaged homogenized continua. This approach avoids the tedious solution of characteristic functions in the conventional asymptotic type method.

An outline of this paper is as follows. In Section 2, the two-scale model problem is stated and the homogenization procedures are defined. Based on the homogenized stress and strain, the energy relation between micro- and macro-scales is established in Section 3. In Section 4, procedures for constructing damage evolution functions based on the cracked microscopic cell solution for a

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scalar damage model, 2-parameter damage model, and fully tensorial damage model are presented. Numerical formulations for the microscopic cell analysis are given in Section 5. In Section 6, the numerical procedures for energy based characterization of the micro-cracks informed damage model in the continuum scale is discussed, and proper scaling of the characterized damage evolution functions to eliminate mesh dependency of the continuum solution is introduced. Concluding remarks are given in Section 7.

2. Model problem and homogenization operators

We start with a two-scale representation of the model problem. A heterogeneous solid with domain Ω and boundary Γ containing a distribution of micro-cracks is considered, as shown in Fig. 1. The solid is subjected to surface traction $\mathbf{t}$ on Γ_t and prescribed displacement $\bar{\mathbf{u}}$ on Γ_u, $\Gamma_t \cup \Gamma_u = \Gamma$, and is assumed to undergo static elastic deformation without body force. For a given material point in the macroscopic solid, it corresponds to a microscopic cell microstructure with domain Ω_y containing micro-cracks with surface, Γ_c. We use $\mathbf{x}$ as the macroscopic coordinate and $\mathbf{y}$ as the microscopic coordinate.

The boundary-value problem is given as follows:

$$\nabla \cdot \boldsymbol{\sigma}^\varepsilon = 0 \quad \text{in } \Omega \tag{1}$$

with the corresponding boundary conditions,

$$\boldsymbol{\sigma}^\varepsilon \cdot \mathbf{n} = \mathbf{t} \quad \text{on } \Gamma_t \tag{2}$$

$$\mathbf{u}^\varepsilon = \bar{\mathbf{u}} \quad \text{on } \Gamma_u \tag{3}$$

and the effective traction acting on the crack surface,

$$\boldsymbol{\sigma}^\varepsilon \cdot \mathbf{n} = \mathbf{h} \quad \text{on } \Gamma_c \tag{4}$$

where $\boldsymbol{\sigma}^\varepsilon$ is the total stress field, $\mathbf{u}^\varepsilon$ is the total displacement field, superscript "ε" denotes that the coarse and fine scale responses are embedded in the total solution, $\mathbf{n}$ is the unit outward normal vector on the boundary, and the traction $\mathbf{h}$ is applied to the union of crack surfaces Γ_c.

We consider a linear elastic material law:

$$\boldsymbol{\sigma}^\varepsilon = \mathbf{C}^\varepsilon : \mathbf{e}^\varepsilon \tag{5}$$

where $\mathbf{C}^\varepsilon$ is the heterogeneous constitutive tensor, and $\mathbf{e}^\varepsilon$ is the total strain field,

$$\mathbf{e}^\varepsilon = \tfrac{1}{2}(\nabla \otimes \mathbf{u}^\varepsilon + \mathbf{u}^\varepsilon \otimes \nabla) \tag{6}$$

Direct simulation of the total scale governing Eqs. (1)–(4) is time consuming due to the fine-scale microscopic defects and heterogeneities, and an attempt is made to conduct homogenization as shown in Fig. 2, where the homogenized stress and strain are defined by the microscopic cell.

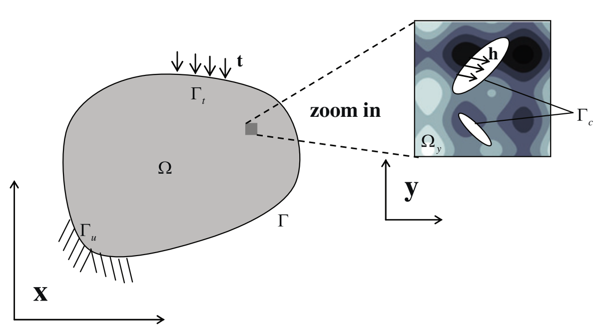

Fig. 1. Microscopic and macroscopic structures.

The tractions and displacements prescribed on the outer boundary of the microscopic cell $\partial\Omega_y$ are related to the homogenized stress and strain in the continuum as:

$$\bar{\boldsymbol{\sigma}} = \frac{1}{V_y} \oint_{\partial\Omega_y} (\mathbf{t}^\varepsilon \otimes \mathbf{x})\, \mathrm{d}s \tag{7}$$

$$\bar{\mathbf{e}} = \frac{1}{2V_y} \oint_{\partial\Omega_y} (\mathbf{u}^\varepsilon \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u}^\varepsilon)\, \mathrm{d}s \tag{8}$$

where $V_y = \int_{\Omega_y} \mathrm{d}\Omega$ is the volume of the microscopic cell. Alternatively, the averaged stress and strain in the microscopic cell are defined as

$$\langle \boldsymbol{\sigma}^\varepsilon \rangle = \frac{1}{V_y} \int_{\Omega_y} \boldsymbol{\sigma}^\varepsilon\, \mathrm{d}\Omega \tag{9}$$

$$\langle \mathbf{e}^\varepsilon \rangle = \frac{1}{V_y} \int_{\Omega_y} \mathbf{e}^\varepsilon\, \mathrm{d}\Omega \tag{10}$$

where

$$\langle \cdot \rangle = \frac{1}{V_y} \int_{\Omega_y} \cdot\, \mathrm{d}\Omega \tag{11}$$

The following equation is used to obtain the relation between homogenized and averaged stresses for the cracked microscopic cell:

$$\nabla \cdot (\boldsymbol{\sigma}^\varepsilon \otimes \mathbf{x}) = \nabla \cdot \boldsymbol{\sigma}^\varepsilon \otimes \mathbf{x} + \boldsymbol{\sigma}^\varepsilon \cdot (\nabla \otimes \mathbf{x}) = \boldsymbol{\sigma}^\varepsilon \tag{12}$$

By substituting Eq. (12) into the averaged stress definition in Eq. (9), we have

$$\begin{aligned} \langle \boldsymbol{\sigma}^\varepsilon \rangle &= \frac{1}{V_y} \int_{\Omega_y} \boldsymbol{\sigma}^\varepsilon\, \mathrm{d}\Omega = \frac{1}{V_y} \int_{\Omega_y} \nabla \cdot (\boldsymbol{\sigma}^\varepsilon \otimes \mathbf{x})\, \mathrm{d}\Omega \\ &= \frac{1}{V_y} \oint_{\partial\Omega_y} (\mathbf{t}^\varepsilon \otimes \mathbf{x})\, \mathrm{d}s - \frac{1}{V_y} \oint_{\Gamma_c} (\mathbf{t}^\varepsilon \otimes \mathbf{x})\, \mathrm{d}s \\ &= \bar{\boldsymbol{\sigma}} - \frac{1}{V_y} \oint_{\Gamma_c} (\mathbf{t}^\varepsilon \otimes \mathbf{x})\, \mathrm{d}s \end{aligned} \tag{13}$$

The second term on the right hand side of Eq. (13) vanishes due to equilibrium of the cohesive stresses on the crack surface. Hence the homogenized stress equals to the averaged stress even in the case of a cracked microscopic cell, that is,

$$\bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma}^\varepsilon \rangle \tag{14}$$

Substituting Eq. (6) into averaged strain defined in Eq. (10) and considering the divergence theorem, we have

$$\begin{aligned} \langle \mathbf{e}^\varepsilon \rangle &= \frac{1}{V_y} \int_{\Omega_y} \mathbf{e}^\varepsilon\, \mathrm{d}\Omega = \frac{1}{2V_y} \int_{\Omega_y} (\nabla \otimes \mathbf{u}^\varepsilon + \mathbf{u}^\varepsilon \otimes \nabla)\, \mathrm{d}\Omega \\ &= \frac{1}{2V_y} \oint_{\partial\Omega_y} (\mathbf{u}^\varepsilon \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u}^\varepsilon)\, \mathrm{d}s - \frac{1}{2V_y} \\ &\quad \times \oint_{\Gamma_c} (\mathbf{u}^\varepsilon \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u}^\varepsilon)\, \mathrm{d}s \\ &= \bar{\mathbf{e}} - \frac{1}{2V_y} \oint_{\Gamma_c} (\mathbf{u}^\varepsilon \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u}^\varepsilon)\, \mathrm{d}s \end{aligned} \tag{15}$$

Consequently we obtain the relation between homogenized strain and averaged strain as follows

$$\bar{\mathbf{e}} = \langle \mathbf{e}^\varepsilon \rangle + \frac{1}{2V_y} \oint_{\Gamma_c} (\mathbf{u}^\varepsilon \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u}^\varepsilon)\, \mathrm{d}s \tag{16}$$

Here it is shown that the homogenized strain consists of the average strain and the additional strain induced by the displacement jump across the crack surfaces. In other words, macroscopic strain cannot be directly obtained by the averaging of microscopic strain when micro-cracks exist.