International Journal of Solids and Structures 48 (2011) 2965–2976

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Analytical and experimental study of a circular membrane in adhesive contact with a rigid substrate

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ARTICLE INFO

Article history:
Received 5 April 2011
Received in revised form 2 June 2011
Available online 28 June 2011

Keywords:
Adhesive contact mechanics
Membranes
Moiré deflectometry
Micro-electro-mechanical systems (MEMS)
Micro-opto-electro-mechanical systems (MOEMS)

ABSTRACT

The problem that is addressed here is that of a pressurized circular membrane in adhesive contact with a rigid substrate. A closed-form membrane analysis is developed for the JKR, DMT and Maugis regimes, which describes the relationships between adhesion energy, pressure, contact radius and contact force. The JKR–DMT transition is studied for this case of membrane contact by introducing an appropriate dimensionless parameter. Experiments are conducted with smooth and structured acrylate layers on a PET carrier film contacting a glass substrate using an apparatus based on moiré deflectometry to measure the contact radius and slope of these thin transparent films. They demonstrate that this analysis predicts the contact radius well. The adhesion energy extracted from the analysis of the measured pressure-contact radius response is constant during unloading but appears to increase during pressurization.

1. Introduction

The adhesion, contact and deformation of thin membranes have played important roles in many fields. For instance, in biological science, cell membrane and substratum adhesion is vital to normal cell functioning and locomotion (Fisher, 1993) and vesicle membrane fusion is of practical importance for targeted drug delivery (Bakowsky et al., 2008). In micro or micro-opto electro-mechanical systems (MEMS or MOEMS), electrostatically driven bridges or diaphragm membranes operate over trillions of cycles in their life span and the study of reliability and durability of such MEMS/MOEMS devices relies on a quantitative understanding and determination of change in adhesion and contact over time (Rebeiz, 2003). Furthermore, an accurate determination of contact size is necessary to evaluate contact resistance, heat dissipation and contact temperature in DC-contact-switch MEMS (Hyman and Mehregany, 1999; Rebeiz, 2003).

The contact mechanics of two elastic solids has been well established through the classical theories of Hertz (1881) , Johnson et al. (1971) (JKR), Derjaguin et al. (1975) and Maugis (1992) (DMT) and Maugis (1992). The valid regimes of these theories are summarized in the Johnson–Greenwood map (Johnson and Greenwood, 1997), which is based on the dimensionless parameter initiated by Tabor (1977). However, these theories for elastic solids are not applicable to thin membranes in contact. This is due to the fact that the elastic strain energy is determined by the membrane stresses which result from the large out-of-plane deflections of the thin membrane. Consequently, geometrical nonlinearity has to be considered and exact closed-form solutions are not possible.

The first configuration that was studied by membrane contact mechanics was a cellular membrane compressed between two parallel plates (Cole, 1932; Harvey, 1938; Hiramoto, 1963). This was used to characterize the mechanical properties of cellular membranes. The other extensively studied configuration is a spherical capsule adhered to a substrate (Shanahan, 1997; Wan and Liu, 2001). This configuration is widely used to explore cell/vesicle/liposome/microcapsule–substrate contact and adhesion which play critical roles in biological and biomedical science. The third class of problems includes one-dimensional strips or axisymmetric membranes contacting a rigid substrate or punch under adhesive surface forces (Nadler and Tang, 2008; Plaut et al., 2001, 1999; Wan, 2002; Wan and Duan, 2002; Wan and Julien, 2009; Wong et al., 2007; Yang, 2004). These configurations have been used to study the contact and adhesion between thin membranes and substrates and stiction and adhesion in biological/biomedical and MEMS structures.

In this paper, a pressurized circular membrane clamped peripherally and contacting a rigid substrate is studied. This can be regarded as a contact configuration in the third category mentioned above. This geometry is also reminiscent of the constrained blister test (Chang et al., 1989; Napolitano et al., 1988) but instead the edge of the blister is clamped and the contact and adhesion between the membrane and the constrained plate

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doi:10.1016/j.ijsolstr.2011.06.014

is of interest. Plaut et al. (2003) obtained extensive numerical solutions for the contact mechanics of this configuration under linear plate, nonlinear plate and membrane assumptions, with or without adhesion. The same author has also studied the mechanical response of axisymmetric membranes with or without contact under various loading conditions and theories (Plaut, 2009a,b). This configuration was also considered in (Flory et al., 2007) where an energy minimization approach was used to account for JKR type adhesion in experiments with three different polymer membranes contacting the gold electrode of a quartz crystal resonator. More recently in (Long et al., 2010) material nonlinearity in the membrane and frictionless and no-slip contact were addressed. A thermodynamic energy balance was used to study the contact and adhesion of a punch test (Li and Wan, 2010a,b; Wan and Julien, 2009). However, residual stresses were not considered in these analyses and so far, no experimental verification of these analyses has been reported. This contact configuration is applicable to the study of the contact and adhesion between thin membranes and substrates in biological/biomedical and MEMS structures. In view of the potential application of these relatively complex analyses to such problems by non-specialists, the development of simpler but accurate closed-form analytical solutions for predicting the relationships between adhesion energy, contact radius, contact force and pressure is attractive. Such solutions may serve the same purpose as the Hertz, DMT, JKR and Maugis contact theories for elastic bodies have provided for many years.

A companion paper has studied the Hertz contact theory of this configuration and the solution was verified by contact experiments using an apparatus based on moiré deflectometry and full numerical solutions (Xu and Liechti, 2010a). In the following, a similar combined analytical and experimental study is pursued. An approximate closed-form analysis that includes residual stresses is developed to predict the relationships between the pressure, contact radius, adhesion energy and contact force using the Dugdale assumption for adhesion interactions (Dugdale, 1960). The resulting membrane counterparts of the classical DMT, JKR and Maugis theories are developed and a transition parameter, similar to the classical Tabor parameter (Tabor, 1977), is introduced to show that the DMT and JKR solutions are two limits of the spectrum covered by the Maugis–Dugdale solution. Finally, this analysis is used to study the contact and adhesion between glass and various samples using an apparatus based on moiré deflectometry (Kafri, 1980; Xu and Liechti, 2010a,b).

This paper is organized as follows: the contact theories and corresponding JKR–DMT transition study are developed in Section 2; this is followed in Section 3 by a summary of the moiré deflectometry apparatus, accompanied by a description of sample preparation, experimental procedures and resulting moiré patterns; in Section 4, the paper concludes with a presentation and discussion of the experimental and analytical results.

2. Analysis

Fig. 1 shows a schematic of the contact configuration for this study. A thin film with a thickness h, Young's modulus E and Poisson's ratio v is clamped peripherally to a substrate with a circular opening of diameter $2a$. A second smooth, frictionless and rigid substrate is placed above the film with an initial gap g. As a pressure p is applied to the thin film, it bulges with a profile $w(r)$ and contacts the upper substrate over a circular region of radius c. The possibility of an equibiaxial residual stress σ_0 in the membrane is considered and the adhesive interactions between the film and the substrate are represented by an energy of adhesion ω. For a given gap, the goal of the present analysis is to find the relationship between the applied pressure, the contact radius, the residual stress, the adhesion energy and the contact force F. The following assumptions are made in order to reach a closed form solution: (i) the thin film has negligible flexural rigidity and only membrane stresses are considered, i.e., a, $g \gg h$; (ii) the gap $g \ll a$ and $sin\theta \approx dw/dr$, where θ is defined in Fig. 1b, thereby placing a restriction on the extent of the contact radius (Xu and Liechti, 2010a); (iii) the radial stress is constant and therefore equal to the hoop stress, i.e., $\sigma_r = \sigma_\theta = \sigma$; (iv) the contact between the film and the rigid surface is frictionless.

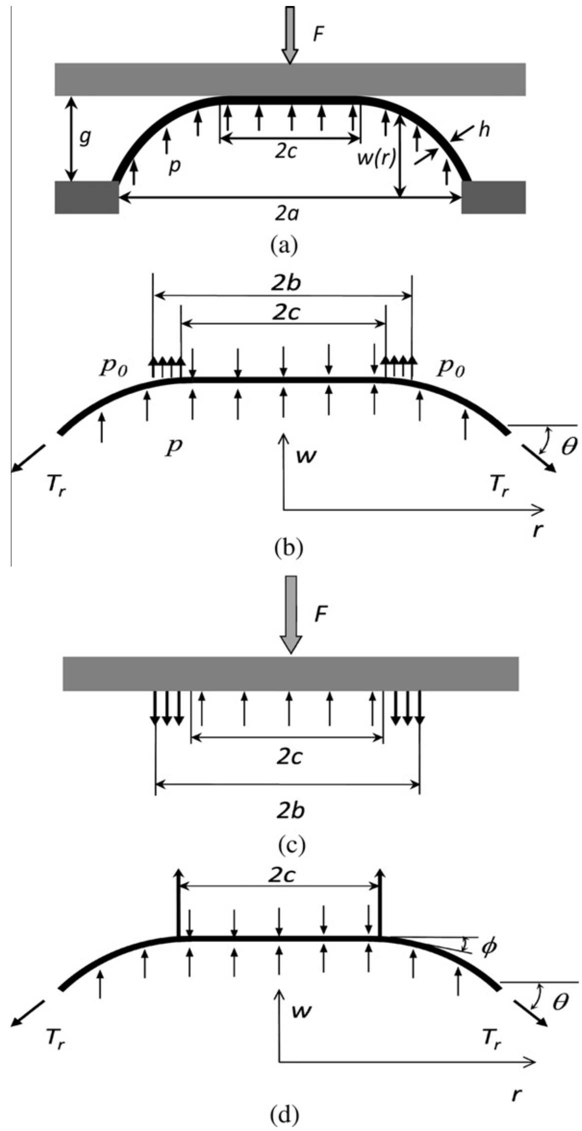

Fig. 1. A schematic of (a) a clamped membrane contacting a rigid plate with an initial gap g between them. The free body diagrams of (b) the membrane and (c) the substrate for the Maugis–Dugdale analysis and (d) the membrane for the JKR analysis.

2.1. Maugis–Dugdale model

A closed form solution, similar to the Maugis–Dugdale model for elastic bodies (Dugdale, 1960; Maugis, 1992), is developed for a constant adhesive traction p_0, with a range of adhesive interactions z_0. In other words, the traction p_0 is active as long as the gap between the interacting surfaces is less than z_0. Under these conditions, the energy of adhesion is

$$\omega = p_0 z_0. \quad (1)$$