International Journal of Solids and Structures 48 (2011) 2102–2119

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

The effect of anisotropic damage evolution on the behavior of ductile and brittle matrix composites

Jacob Aboudi *

Faculty of Engineering, Tel-Aviv University, Ramat-Aviv 69978, Israel

ARTICLE INFO

Article history:
Received 30 August 2010
Received in revised form 1 February 2011
Available online 23 March 2011

Keywords:
Continuum damage mechanics
Damage evolution law
Anisotropic damage
Full thermomechanical coupling
High fidelity generalized method of cells

ABSTRACT

Anisotropic damage evolution laws for ductile and brittle materials have been coupled to a micromechanical model for the prediction of the behavior of composite materials. As a result, it is possible to investigate the effect of anisotropic progressive damage on the macroscopic (global) response and the local spatial field distributions of ductile and brittle matrix composites. Two types of thermoinelastic micromechanics analyses have been employed. In the first one, a one-way thermomechanical coupling in the constituents is considered according to which the thermal field affects the mechanical deformations. In the second one, a full thermomechanical coupling exists such that there is a mutual interaction between the mechanical and thermal fields via the energy equations of the constituents. Results are presented that illustrate the effect of anisotropic progressive damage in the ductile and brittle matrix phases on the composite's behavior by comparisons with the corresponding isotropic damage law and/or by tracking the components of the damage tensor.

1. Introduction

In the most general formulation of the continuum damage mechanics, the damage state should be represented by a fourth-order tensor. Such a formulation however would be too difficult and not necessary, Voyiadjis and Kattan (2005) and Lemaitre and Desmorat (2005). Theories with scalar damage variables are the easiest to handle. Anisotropic damage theories are based on second-order damage state representations. Discussions and presentations of continuum damage mechanics and the various formulations of the damage states and their evolution laws can be found in the books by Kachanov (1986), Lemaitre and Chaboche (1990), Krajcinovic (1996), Lemaitre (1996), Voyiadjis and Kattan (2005), and Lemaitre and Desmorat (2005), for example. Applications of continuum damage mechanics theories on composites materials are given by Talreja (1985a,b, 1994), Voyiadjis and Deliktas (1997) Voyiadjis and Kattan (1999), Skrzypek and Ganczarski (1999), Barbero (2008), Haj-Ali (2009), Bednarcyk et al. (2010), Haj-Ali and Aboudi (2010) and references cited there.

Lemaitre et al. (2000) presented a continuum damage theory with anisotropic damage evolution in ductile materials that generalizes the isotropic damage theory of Lemaitre (1985a,b) which is based on a scalar variable. In addition, Lemaitre and Desmorat (2005) presented anisotropic damage model for a brittle material (concrete). The purpose of the present investigation is to couple these theories with a micromechanics model for the prediction of the response of ductile and brittle matrix composites with evolving damage. As is shown in this investigation, the resulting micromechanics analyses enable the study of the effect of anisotropic damage laws by comparisons with the corresponding isotropic damage theories and by tracking the evolutions of the components of the damage tensor.

The micromechanics model that is employed in the present investigation is referred to as *The High Fidelity Generalized Method of Cells* (HFGMC) which is based on the homogenization procedure for periodic multiphase composites. This micromechanics model has been reviewed by Aboudi (2004) and more recently has been shown by Haj-Ali and Aboudi (2009) to provide excellent prediction for (undamaged) nonlinear and inelastic matrix composites by extensive comparisons with finite element solutions. In addition, this micromechanics model has been coupled by these authors to a finite element software to investigate the response of metal matrix composite structures.

This paper is organized as follows. In Section 2, the isotropic and anisotropic damage theories in unreinforced ductile materials are presented. This is follows by the presentation of the anisotropic damage theory of a brittle material. In Section 3, the HFGMC micromechanics theory is outlined. This includes the one-way and fully coupled thermoinelastic HFGMC. In the former theory, the conventional constitutive equations is employed according to which the thermal effects in the constituent affect the mechanical response of the material. In the latter theory, the fully coupled

* Tel.: +972 3 640 8131; fax: +972 3 640 7617.
E-mail address: aboudi@eng.tau.ac.il

0020-7683/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2011.03.014

HFGMC is discussed according to which, a mutual interaction exists between the mechanical and temperature fields in the constituents (Aboudi, 2008). This mutual interaction between the mechanical and thermal effects is governed by the coupled energy equation of the constituents. Due to the inelasticity effects in the metallic matrix, a major part of the rate of plastic work is liberated as a heat source to be included in the energy equation. The most interesting result from the fully coupled HFGMC theory are the spatial temperature distributions in the composite which are induced by the externally applied mechanical loadings. The generation of these temperature distributions enables the identification of critical hot spots in the composite caused by the mechanical loading. These hot spots indicate the existence of high inelastic strains which my lead to ultimate failure. In Section 4, extensive comparisons between the effects of anisotropic and isotropic damage laws in the ductile phase of metal matrix composites are presented. For the brittle matrix composite, the effect of anisotropic damage can be evaluated by tracking the evolution of the components of the damage tensor. Finally, a Conclusion section discusses possible future investigations.

2. Constitutive equations of the monolithic materials with progressive damage

2.1. Ductile materials with isotropic evolving damage

For thermoelastoplastic materials, the total strain tensor is decomposed, in the framework of the infinitesimal strain theory, into elastic, thermal and plastic components in the form:

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^t + \boldsymbol{\epsilon}^p \tag{1}$$

The constitutive equations of these materials with isotropic damage law can be determined from the Gibbs potential G (per unit volume) as follows

$$\boldsymbol{\epsilon} = \frac{\partial G}{\partial \boldsymbol{\sigma}} = \frac{\partial G_{et}}{\partial \boldsymbol{\sigma}} + \boldsymbol{\epsilon}^p \tag{2}$$

where $\boldsymbol{\sigma}$ is the stress tensor and G_{et} is the thermoelastic portion of G. The expression for the thermoelastic contribution G_{et} is given by (Lemaitre and Desmorat, 2005):

$$G_{et} = \frac{1+\nu}{2E}\frac{\boldsymbol{\sigma}:\boldsymbol{\sigma}}{1-D} - \frac{\nu}{2E}\frac{tr^2(\boldsymbol{\sigma})}{1-D} + \alpha(T-T_0)tr(\boldsymbol{\sigma}) \tag{3}$$

where $tr(\boldsymbol{\sigma})$ is the trace of the stress tensor, $\boldsymbol{I}$ is the identity second-order tensor, $0 \leqslant D \leqslant 1$ is the damage variable, $T - T_0$ is the temperature deviation from a reference temperature T_0 and E, ν and α are the Young's modulus, Poisson's ratio and coefficient of thermal expansion of the isotropic material. Consequently, the following expression for the elastic and thermal strains is obtained

$$\boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^t = \frac{\partial G_{et}}{\partial \boldsymbol{\sigma}} = \frac{1+\nu}{E}\tilde{\boldsymbol{\sigma}} - \frac{\nu}{E}tr(\tilde{\boldsymbol{\sigma}})\boldsymbol{I} + \alpha(T-T_0)\boldsymbol{I} \tag{4}$$

with $\tilde{\boldsymbol{\sigma}}$ being the effective stress which is related to the stress $\boldsymbol{\sigma}$ in the form: $\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/(1-D)$. This equation provides the following expression for the stress and effective stress tensors:

$$\boldsymbol{\sigma} = (1-D)\boldsymbol{h} : \boldsymbol{\epsilon}^e \tag{5}$$

and

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{h} : \boldsymbol{\epsilon}^e \tag{6}$$

with $\boldsymbol{h}$ being the standard forth-order stiffness tensor of isotropic materials

$$\boldsymbol{h} = \lambda \boldsymbol{I} \otimes \boldsymbol{I} + 2\mu \boldsymbol{I}_4 \tag{7}$$

where λ and μ are the Lame' constants and $\boldsymbol{I}_4$ is the forth-order unit tensor. It should be noted that Eq. (6) is based on the principle of strain equivalence according to which the strains in the damaged and effective configurations are equal. Hence, the strain constitutive equations of the damaged material are derived from the corresponding equations of the undamaged material by replacing stress in the latter by the equivalent stress.

By assuming isotropic hardening, the yield function ϕ is given by

$$\phi = \tilde{\sigma}_{eq} - \sigma_y = \frac{1}{1-D}\sigma_{eq} - \sigma_y \tag{8}$$

where X_{eq} of the second-order tensor $\boldsymbol{X}$ stands for

$$X_{eq} = \sqrt{\frac{3}{2}dev(\boldsymbol{X}) : dev(\boldsymbol{X})} = \sqrt{\frac{3}{2}}\|dev(\boldsymbol{X})\| \tag{9}$$

and $dev(\boldsymbol{X})$ is the deviator of $\boldsymbol{X}$. In Eq. (8), σ_y is the function that describes the hardening of the elastoplastic material. For isotropic hardening it is given by

$$\sigma_y = Y_0 + K(R) \tag{10}$$

where Y_0 is the yield stress in simple tension and $K(R)$ describes the isotropic hardening law. For linear hardening: $K(R) = H_0 R$. The rate of hardening is given by

$$\dot{R} = -\dot{\gamma}\frac{\partial \phi}{\partial K} = \dot{\gamma} \tag{11}$$

where γ is the consistency parameter. The evolution of the plastic strains is given by

$$\dot{\boldsymbol{\epsilon}}^p = \dot{\gamma}\frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \frac{3\dot{\gamma}}{2(1-D)}\frac{dev(\tilde{\boldsymbol{\sigma}})}{\tilde{\sigma}_{eq}} = \sqrt{\frac{3}{2}}\frac{\dot{\gamma}}{1-D}\frac{dev(\boldsymbol{\sigma})}{\|dev(\boldsymbol{\sigma})\|} \tag{12}$$

The equivalent plastic strain can be obtained from this equation as follows

$$\dot{\epsilon}_p = \sqrt{\frac{2}{3}}\|\dot{\boldsymbol{\epsilon}}^p\| = \frac{\dot{\gamma}}{1-D} \tag{13}$$

The energy release rate Y is given in the form (Lemaitre and Desmorat, 2005)

$$\begin{aligned} Y &= -\frac{1}{2}\boldsymbol{\epsilon}^e : \boldsymbol{h} : \boldsymbol{\epsilon}^e = -\frac{\tilde{\sigma}_{eq}^2}{2E}\left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] \\ &= -\frac{\sigma_{eq}^2}{2E(1-D)^2}\left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] \end{aligned} \tag{14}$$

where $\sigma_H = tr(\boldsymbol{\sigma})/3$ being the hydrostatic stress.

Finally, the dissipation function ψ, also referred to as inelastic potential function, from which the inelastic flow rule and the evolution laws for the internal variables and damage are derived. It is taken in the form

$$\psi = \phi + \frac{S}{(1-D)(s+1)}\left(\frac{-Y}{S}\right)^{s+1} \tag{15}$$

where S and s are material constants. This function provides the isotropic evolution law of damage in the form

$$\dot{D} = -\dot{\gamma}\frac{\partial \psi}{\partial Y} = \frac{\dot{\gamma}}{1-D}\left(\frac{-Y}{S}\right)^s \tag{16}$$

The above system of nonlinear equations together with the condition that $\phi = 0$ that govern the behavior of monolithic thermoelastolplastic materials with isotropic evolving damage are solved incrementally in conjunction with the return mapping algorithm (de Souza Neto et al., 2008).