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An incremental variational formulation of dissipative magnetostriction at the macroscopic continuum level

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ABSTRACT

This paper outlines a new variational-based modeling and computational implementation of macroscopic continuum magneto-mechanics involving non-linear, inelastic material behavior, with a special focus on dissipative magnetostriction. It is based on a *constitutive variational principle* that optimizes a generalized incremental work function with respect to the internal state variables. In an incremental setting at finite time steps, this variational problem defines a *quasi-hyper-magnetoelastic potential* for the stresses and the magnetic induction, and incorporates energy storage as well as dissipative mechanisms. The existence of this potential further allows the incremental boundary-value problem of quasi-static inelastic magneto-mechanics to be recast into a *principle of stationary incremental energy*. The second focus of this paper is on the careful construction of the energy storage and dissipation functions for the model problem of hysteretic magnetostriction at the macroscopic level. It is then demonstrated that the proposed model is capable of predicting the ferromagnetic and field-induced strain hysteresis curves characteristic of magnetostrictive material response in good agreement with experiments. The numerical solution of the coupled non-linear boundary-value problem is based on a monolithic multi-field finite element implementation. As a consequence of the proposed incremental variational principle, the discretization of the multi-field problem appears in a *compact symmetric format*. In this sense, the proposed formulation provides a canonical framework for the simulation of boundary-value-problems in dissipative magnetostriction at the macro-level. The performance of the proposed algorithm is tested by application to relevant numerical examples.

1. Introduction

Magnetostrictives can be classified as *active materials*, which by definition, exhibit an intrinsic coupling of mechanical output with non-mechanical input. A wide range of applications of bulk, thin film or multi-layer magnetostrictives has been suggested and realized, which make use of the intrinsic sensing and actuation capabilities. These include ultrasonic transducers for sonar devices, shock and vibration absorbers, linear motors, high precision positioning devices, broadband seismic energy sources for geological imaging, thin film ultrasonic motors, micro-valves and pumps (Trémolet de Lacheisserie, 1993; Engdahl, 2000). The recent resurgence of interest in magnetostrictive materials is primarily explained by the commercial availability of so-called *giant magnetostrictive materials*. The rare earth-iron intermetallic compound Terfenol-D[1] ($Tb_xDy_{1-x}Fe_{2-y}$, with $x = 0.3$, $0 \leqslant y \leqslant 0.2$) may be considered the most technologically-advanced magnetostrictive material. It produces quasi-static field-induced strains over 0.16%, i.e. two orders of magnitude larger strains than naturally-magnetostrictive elements such as iron, in response to moderate magnetic fields of less than 160 kA/m (Moffet et al., 1991; Dapino et al., 2006). Galfenol ($Fe_{1-x}Ga_x$, with $13 \leqslant x \leqslant 23$ at. %), in which up to 400 ppm (or 0.04%) may be induced at room temperature, has also been widely investigated (Kellogg et al., 2002).

1.1. Modeling approaches to dissipative magnetostriction

Typical macroscopic magneto-mechanical response curves are displayed in Fig. 1. One observes the non-linear and dissipative nature of the characteristic "butterfly" field-induced strain and ferromagnetic hysteresis curves. Note that these macroscopic response curves are qualitatively very similar to the dielectric and strain hystereses exhibited by ferroelectric ceramics (see e.g. Zhou et al., 2001), although the basic physical mechanisms are fundamentally different. For some applications, however, magnetostrictive materials possess properties that are superior to those of piezo-ceramics with respect to their thermodynamic stability.

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[1] The suffix -nol stands for the US Naval Ordnance Laboratory, where these materials were originally developed in the 1970s.

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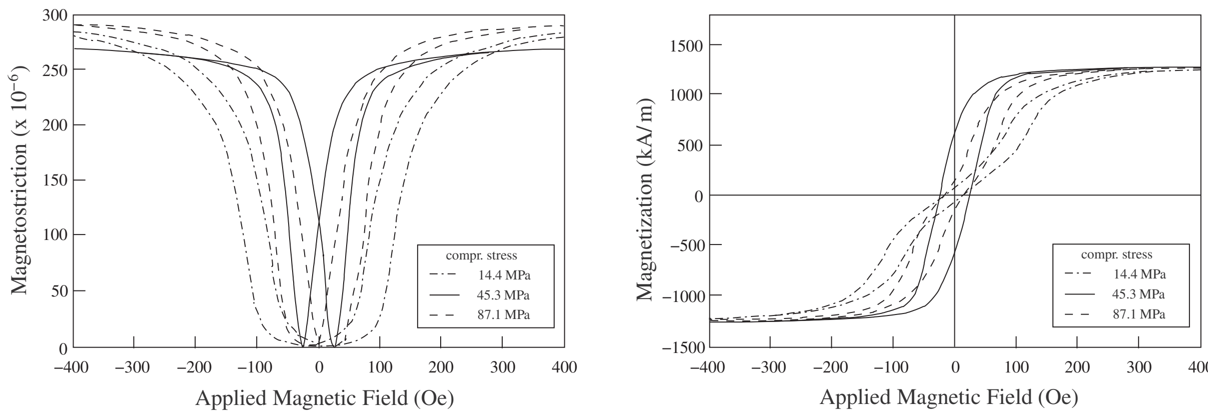

Fig. 1. Magnetostriction and magnetization curves for $Fe_{0.81}Ga_{0.19}$ (Galfenol) at constant temperature for various compressive stresses, adapted from Kellogg et al. (2002).

Terfenol-D, for example, possesses a working range of −15 °C to 70 °C. Piezoceramics, on the other hand exhibit reduced coupling properties even at 50% of the Curie temperature (Nobili and Tarantino, 2008).

Pioneering work in establishing a theoretical framework of *the continuum mechanics of polarizable and magnetizable matter* was presented by Landau and Lifshitz (1960), Truesdell and Toupin (1960), Hutter and van de Ven (1978) and Eringen and Maugin (1990); see also the overview given in Kovetz (2000) and the references therein. As far as the constitutive model formulation for non-linear, dissipative magnetostriction is concerned, two *fundamentally different* approaches can be found in the literature. The formation and evolution of *magnetic microstructures* under the influence of mechanical and magnetic fields is classically modeled based on the variational *theory of micromagnetics*, originally proposed by Brown (1963, 1966). For a detailed discussion of the mathematical foundations and recent developments in computational micromechanics, the reader is also referred to DeSimone et al. (2004). A severe short-coming of such theories, however, is due to the fact that physically-meaningfull predictions require the resolution of complex, three-dimensional microstructures. In consequence, it is not feasible to employ such simulations to samples or even full-scale devices of technologically-relevant size. In order to alleviate this difficulty, but preserve the micromechanical spirit of the modeling approach, DeSimone and James (2002) developed a mathematically-rigorous specialization of the micromagnetic theory, the so-called *constrained theory of magnetoelasticity*. It describes the formation of fine-scale microstructures based on the relaxation of non-convex energy landscapes, where the space of admissible configurations is severely reduced by the assumptions of high anisotropy and the exchange energy is neglected in the large body limit. Although this micromechanically-motivated model has been shown to qualitatively predict important features of macroscopic material response, its simplyfing assumptions are quite severe and the characterisitic dissipative effects cannot be accounted for.

On the other hand, *continuum mechanics-based macroscopic models*, to which we shall restrict our attention in this paper, have proven very successful in predicting the characteristic non-linear, hysteretic and anisotropic effective properties of *macroscopic* magnetostrictive material response. A thermodynamically-consistent, non-linear, non-dissipative model was presented by Carman and Mitrovic (1995). Fang et al. (2004) presented a model, in which the remanent strain and remanent magnetization are treated as internal variables. This formulation may be viewed as a generalization of classical associative plasticity, in that a quadratic yield-surface with kinematic hardening is introduced in stress-magnetic field space and evolution equations for the internal state variables follow from a generalized normality rule. Linnemann et al. (2009) recently presented a thermodynamically motivated constitutive model for magnetostrictive and piezoelectric materials as an extension of their earlier work on ferroelectrics, see e.g. Klinkel (2006). In this approach a specific free energy function and a switching criterion are proposed. Furthermore, for this internal variable model an additive split of the strains and the magnetic field strength, not the magnetization, is suggested. An overview of similar modeling approaches for the *magnetic shape memory effect*, which has often been referred to as the *magnetostriction of martensite*, can for example be found in Kiang and Tong (2005) and Kiefer and Lagoudas (2009).

Though many fundamental concepts for the modeling of hysteresis effects were originally developed for dissipative ferromagnetism, most of the recent work on the modeling of inelastic coupling effects in active solids has been concerned with the prediction of piezo-electric and ferroelectric material behavior, which, as mentioned, exhibits many of the same key macroscopic response characteristics observed in magnetostrictives. The reader is referred to Bassiouny et al. (1988), Cocks and McMeeking (1999), Kamlah and Böhle (2001), Kamlah and Tsakmakis (1999), Landis (2002), and Schröder and Romanowski (2005) for a brief overview of phenomenological, non-linear continuum mechanics-based models for ferroelectrics. In particular, a rigorous formulation of variational principles which govern the coupled boundary-value-problem of dissipative magnetostriction at the macroscopic level is missing in the literature.

1.2. A variational approach to macroscopic magnetostriction

In this paper, a variational-based modeling and computational implementation of dissipative magnetostriction at the macroscopic level is presented, which allows the quantitatively-accurate description of the coupled, non-linear and inelastic response of dissipative magnetostrictive materials. The novel contribution of this work is the variational setting of the formulation, in which updates of the constitutive and the primary field variables are computed on the basis of *local* and *global variational minimization and stationarity principles*. In contrast to *absolute* energy minimization concepts, used in the theory of micromagnetics by Brown (1963, 1966) and DeSimone and James (2002), we propose a *rate-type, incremental* variational principle for the macroscopic description of dissipative magnetostriction. It must be emphasized that this approach is fundamentally different in nature to the approach suggested by Linnemann et al. (2009), where an extended virtual work principle was used to derive the weak form for the coupled boundary value problem. In the formulation proposed in this paper, we define a *constitutive minimization principle* for inelastic magneto-mechanical