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On the limit velocity and buckling phenomena of axially moving orthotropic membranes and plates

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ABSTRACT

In this paper, we consider the static stability problems of axially moving orthotropic membranes and plates. The study is motivated by paper production processes, as paper has a fiber structure which can be described as orthotropic on the macroscopic level. The moving web is modeled as an axially moving orthotropic plate. The original dynamic plate problem is reduced to a two-dimensional spectral problem for static stability analysis, and solved using analytical techniques. As a result, the minimal eigenvalue and the corresponding buckling mode are found. It is observed that the buckling mode has a shape localized in the regions close to the free boundaries. The localization effect is demonstrated with the help of numerical examples. It is seen that the in-plane shear modulus affects the strength of this phenomenon. The behavior of the solution is investigated analytically. It is shown that the eigenvalues of the cross-sectional spectral problem are nonnegative. The analytical approach allows for a fast solver, which can then be used for applications such as statistical uncertainty and sensitivity analysis, real-time parameter space exploration, and finding optimal values for design parameters.

1. Introduction

As is well known, the mechanical behavior of paper under a non-failure condition is adequately described by the model of an elastic orthotropic plate. The rigidity coefficients of the plate model, describing the tension and bending of the paper sheet, have been estimated for various types of paper in many publications (see, for example, Göttsching and Baumgarten, 1976; Thorpe, 1981; Skowronski and Robertson, 1985; Seo, 1999). The deformation properties of a sheet of paper under tensile stress or strain are used in simulation of axial movement of a paper web. In particular, these properties are important for modeling the instability of the axially moving elastic web.

There are numerous studies on the loss of stability of moving elastic webs based on one-dimensional models, using second- and fourth-order differential equations. These studies are devoted to various aspects of free and forced vibrations, including the nature of wave propagation in moving media, and the effects of axial motion on the frequency spectrum and eigenfunctions. The studies include e.g., Archibald and Emslie (1958); Miranker (1960); Swope and Ames (1963); Mote (1968, 1972, 1975); Simpson (1973); Ulsoy and Mote (1980, 1982); Chonan (1986), and Wickert and Mote (1990).

Two-dimensional studies have also been performed. Lin and Mote (1995) studied an axially moving membrane in a 2D formulation, predicting the equilibrium displacement and stress distributions under transverse loading. Later, the same authors predicted the wrinkling instability and the corresponding wrinkled shape of a web with small flexural stiffness (Lin and Mote, 1996). The stability and vibration characteristics of an axially moving plate were investigated by Lin (1997). The loss of stability was studied with application of dynamic and static approaches and the approach by Wickert and Mote (1990) to derive the equation of motion for the plate in matrix form and to use the Galerkin method. It was shown by means of numerical analysis that, for all cases dynamic instability (flutter) is realized when the frequency is zero and the critical velocity coincides with the corresponding velocity obtained from static analysis. In Shin et al. (2005), the out-of-plane vibration of an axially moving membrane was studied. Also here, it was found by numerical analysis, that for a membrane with a no-friction boundary condition in the lateral direction along the rollers, the membrane remains dynamically stable until the critical speed, at which statical instability occurs.

The two-dimensional problem of instability analysis of an axially moving elastic plate was formulated and investigated analytically in Banichuk et al. (2010) in the context of the isotropic model. It was observed that the transverse deflection localizes near the free edges. This was noted to correspond to the eigenfunctions of

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stationary plates under in-plane compressive load (see, e.g., Gorman, 1982), and the results matched the above mentioned results of Lin (1997) and Shin et al. (2005) for moving plates and membranes.

Hatami et al. (2009) studied free vibration of the moving orthotropic rectangular plate in sub- and supercritical speeds, and flutter and divergence instabilities at supercritical speeds. Their study was limited to simply supported boundary conditions at all edges. Free vibrations of orthotropic rectangular plates, which are not moving, have been studied by Biancolini et al. (2005) including all combinations of simply supported and clamped boundary conditions on the edges. Xing and Liu (2009) obtained exact solutions for free vibrations of stationary rectangular orthotropic plates considering three combinations of simply supported (S) and clamped (C) boundary conditions: SSCC, SCCC and CCCC. Kshirsagar and Bhaskar (2008) studied vibrations and buckling of loaded stationary orthotropic plates. They found critical loads of buckling for all combinations of boundary conditions S, C and F.

In this paper, the analysis from Banichuk et al. (2010) is generalized to the case of orthotropic material. Paper materials are an example of this, since they have a fiber structure which can be described as orthotropic on the macroscopic level. To understand the behavior of such materials, it is thus important to consider in which ways and how much the introduction of the orthotropic material model changes the predictions with respect to those from the isotropic model. We develop an analytical solution for the orthotropic case, analogous to the isotropic development in our earlier study. Only a transcendental equation needs to be solved numerically. This allows for a fast solver, which can then be used for applications such as statistical uncertainty and sensitivity analysis, real-time parameter space exploration, and finding optimal values for design parameters.

The problem of stability of an axially moving orthotropic elastic plate traveling between two rollers at a constant velocity, and experiencing small transverse vibrations, is considered in a two-dimensional formulation. The model of a thin elastic orthotropic plate, which is subjected to bending and tension, is used for describing the bending moment and the distribution of membrane forces. The static form of instability is investigated and the critical regime is studied as a function of geometric parameters and the moduli of orthotropicity. It is shown that for some values of the problem parameters, the buckling mode becomes localized in the vicinity of the free boundaries also for the orthotropic plate.

It is well known that by using Huber's estimate for the shear modulus of an orthotropic plate, the equations reduce to those of the isotropic case. In this paper, it is briefly noted that this approach works also for the time-dependent dynamic equation of the moving plate. However, in the analysis proper, we will not make this assumption, but instead develop the analysis for the general orthotropic case.

Huber's estimate is not the only way to theoretically estimate the shear modulus. Methods such as calculating the Young modulus in the 45° direction based on laminate theory, and Campbell's estimate have been used (Yokoyama and Nakai, 2007). Thus, if we allow for the use of actual measured values for the shear modulus, it is possible to compare the different estimates. Also, this makes possible the sensitivity analysis of the shear modulus with respect to any deviation from the Huber estimate.

It should be noted that when applied to the particular context of paper production, this study still ignores some potentially important effects. First, damping effects resulting from the viscoelastic nature of paper are neglected. This is not a major problem, as the introduction of damping is not expected to change the critical velocity, although it does modify the postdivergence behavior (Ulsoy and Mote, 1982). Secondly, the interaction between the traveling web and the surrounding air is known to influence the critical velocity (Pramila, 1986; Frondelius et al., 2006) and the dynamical response (Kulachenko et al., 2007), possibly also affecting the buckling shape. These effects are ignored by the in-vacuum model used in the present study.

2. Basic relations for transverse vibrations of an axially moving orthotropic band

Consider an elastic band traveling with a constant velocity V_0 in the x direction between two rollers located at $x = 0$ and $x = \ell$ in a Cartesian coordinate system. Fig. 1 shows a rectangular part of the band

$$\Omega : 0 \leqslant x \leqslant \ell, \quad -b \leqslant y \leqslant b,$$

where ℓ and b are prescribed parameters. Assume that the considered part of the band has constant thickness h.

Additionally, assume that the band is represented as a rectangular elastic orthotropic plate having bending rigidities D_1, D_2 and D_3, or as a rectangular orthotropic membrane with zero bending rigidities. The "1" axis of the orthotropic material is aligned with the x direction, while the "2" axis is aligned with the y direction (see Fig. 1). The band is subjected to homogeneous tension, acting in the x direction. The sides of the band $x = 0$, $-b \leqslant y \leqslant b$ and $x = \ell$, $-b \leqslant y \leqslant b$ are simply supported, and the sides $y = -b$, $0 \leqslant x \leqslant \ell$ and $y = b$, $0 \leqslant x \leqslant \ell$ are free of tractions.

The transverse displacement of the traveling band is described by the deflection function w, which depends on the coordinates x, y and time t. The differential equation for small transverse vibrations has the form

$$m\left(\frac{\partial^2 w}{\partial t^2} + 2V_0\frac{\partial^2 w}{\partial x \partial t} + V_0^2\frac{\partial^2 w}{\partial x^2}\right) = T_{xx}\frac{\partial^2 w}{\partial x^2} + 2T_{xy}\frac{\partial^2 w}{\partial x \partial y} + T_{yy}\frac{\partial^2 w}{\partial y^2} - \mathcal{L}(w), \tag{1}$$

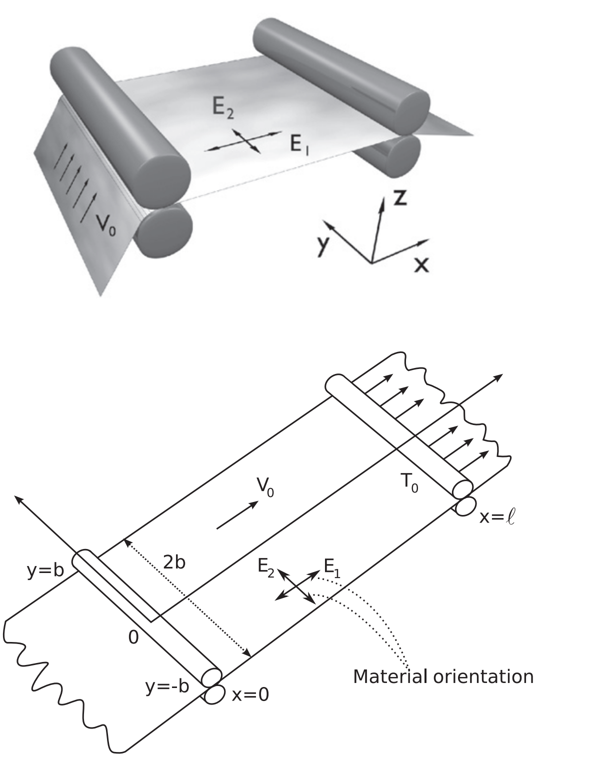

Fig. 1. Axially moving elastic orthotropic band, simply supported at $x = 0$ and $x = \ell$.