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The dispersion of shear wave in multilayered magnetoelastic self-reinforced media

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ABSTRACT

In this paper, we study the propagation of shear waves in a magnetoelastic self-reinforced medium using finite difference technique. Dispersion equation has been deduced for the case when $(n-1)$ layers lie over a half space. It is observed that the obtained dispersion equation is in assertion with the classical Love wave equation for both the cases when a single and double layer lies over a half space. The stability condition for the used finite difference scheme and the expression for the phase and group velocity have been derived. The dispersion curve for different values of magnetoelastic coupling parameter, phase and group velocity variation for different values of stability ratio has been depicted by means of graphs.

1. Introduction

The wave propagation in a reinforced media plays a vital role in construction sector and geophysics. The characteristic property of a self-reinforced material is that its component act together as a single anisotropic unit as long as they remain in elastic condition, i.e. the two components are bound together so that there is no relative displacement between them. There is sufficient evidence in the literature that the Earth crust may contain some hard/soft rocks or material that may exhibit self-reinforcement properties. These rocks when they come in the way of seismic waves do effect their propagation and such seismic signals are always influenced by the elastic properties of media through which they travel. Anderson (1962) has studied Love wave propagation in a medium composed of transversely isotropic layers. He has solved the boundary value problem for a simple layer and extended to multilayered media by a generalization of Haskell's technique.

The idea of introducing a continuous reinforcement at every point of an elastic solid was given by Belfield et al. (1983). Later Verma and Rana (1983) applied this model to the rotation of tube, illustrating its utility in strengthening the lateral surface of the tube. Verma (1986) also discussed the propagation of magnetoelastic shear waves in self-reinforced bodies. The problem of magnetoelastic transverse surface waves in self-reinforced elastic solids was studied by Verma et al. (1988). Chattopadhyay and Chaudhury (1990) studied the propagation, reflection and transmission of magnetoelastic shear waves in a self-reinforced elastic medium. Chattopadhyay and Chaudhury (1995) studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. Chattopadhyay and Venkateswarlu (1998) investigated a two-dimensional problem of stress produced by a pulse of shearing force moving over the boundary of a fibre-reinforced medium. Various approaches for characterizing the effective thermal, electrical, and elastic properties of composite media have been introduced since Maxwell's seminal work on spherical particle suspensions more than a century ago (Milton, 2002). Chaudhary et al. (2005) studied transmission of shear waves through a self-reinforced layer between two inhomogeneous elastic half-spaces. Chaudhary et al. (2006) also discussed the plane SH wave response from elastic slab interposed between two different self-reinforced elastic solids. Recently, Chattopadhyay et al. (2009) have shown that the propagation of Torsional waves in fibre-reinforced material is also possible.

Most of the techniques for solving the wave equation involve some sort of approximation as the reflectivity method assumes lateral homogeneity, Ray method assumes that the seismic wavelength is short compared with the scale of heterogeneity. But the finite element method and finite difference method (Alterman and Karal, 1968; Alford et al., 1974; Kelly et al., 1976) solved the exact elastic wave equation for generally heterogeneous media. This paper deals only with the finite difference method. Finite difference method (FDM) is preferred because of its power, accuracy, reliability, rapidity and flexibility. Early researches on finite difference methods for seismic forward modelling include the work of Alterman and Karal (1968) and Kelly et al. (1976). Earlier Thompson (1950) and Haskell (1953) have given a matrix formalism to obtain the phase velocity dispersion equation for Rayleigh and Love-type elastic surface wave in solid multilayered media. Later Crampin (1970) extended the matrix procedure to obtain the dispersion relation for generalized surface waves in multilayered

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media, where the layer may be either isotropic or anisotropic. Kalyani et al. (2008) have solved a problem of wave propagation in monoclinic media using finite difference method.

When the wave equation is solved in the frequency domain, the accuracy requirement is the most important but when the wave equation is solved in the time domain, it is necessary to achieve both accuracy and stability requirements. By considering higher-order schemes, accuracy problem can be tackled, but a numerical scheme must meet the stability requirement. Many researchers (e.g. Mitchell, 1969; Gazdag, 1981; Holberg, 1987; Cao and Greenhalgh, 1998) have developed stability criteria for various numerical schemes for 1-D, 2-D and 3-D wave simulation.

In this paper, we have studied the propagation of Shear waves in multilayered magnetoelastic self-reinforced medium by explicit finite difference technique and deduced the general dispersion relation considering $(n-1)$ layers lying over a half space. It has been observed that the obtained dispersion equation reduces to the classical SH wave equation for the isotropic case when a single or double layer is considered over a half space. It is distinctly marked that phase velocity dispersion curve is affected by magnetoelastic self-reinforced parameter. The stability criterion for the used finite difference scheme and the expression in terms of stability ratio (Aki and Richards, 1980) for the phase and group velocity has been derived. The phase/group velocities have been computed for different values of stability ratio and are presented by means of graphs.

2. Formulation and solution of the problem

Let us consider total $(n-1)$ magnetoelastic self-reinforced layer lying over a magnetoelastic self-reinforced half-space. The x-axis has been taken along the propagation of waves and z-axis is positive vertically downwards as shown in Fig. 1. The numbering of the layers is being done in top down fashion, i.e. the uppermost layer is numbered as (1) and half space as (n). At first, we need to find the equation governing the propagation of SH wave in self-reinforced magnetoelastic crustal layer.

The constitutive equations used in a self-reinforced linearly elastic model are (Belfield et al., 1983)

$$\begin{aligned}\tau_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) \\ &\quad + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j, \\ &\quad i,j,k,m = 1,2,3,\end{aligned} \tag{1}$$

where τ_{ij} are components of stress, e_{ij} components of infinitesimal strain, δ_{ij} Kronecker delta, a_i components of $\vec{a}$, all referred to rectangular cartesian co-ordinates x_i. $\vec{a} = (a_1, a_2, a_3)$ is the preferred directions of reinforcement such that $a_1^2 + a_2^2 + a_3^2 = 1$. The vector $\vec{a}$ may be function of position. Indices take the values 1, 2, 3 and summation convention is employed. The coefficients λ, μ_T, α, β and $2(\mu_L - \mu_T)$ are elastic constants with dimension of stress. μ_T can be identified as the shear modulus in transverse shear across the preferred direction, and μ_L as the shear modulus in longitudinal shear in the preferred direction. α and β are specific stress components to take into account different layers for concrete part of the composite material. The model considered here is of transversely isotropic material, also known as materials of hexagonal symmetry.

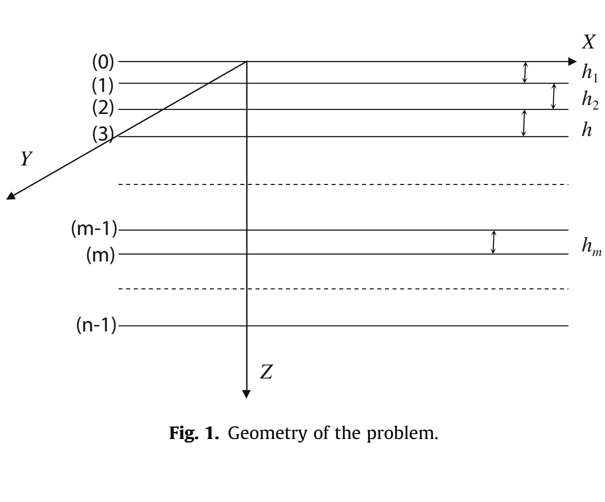

Fig. 1. Geometry of the problem.

Equations governing the propagation of small elastic disturbances in a perfectly conducting self-reinforced elastic medium having electromagnetic force $\vec{J} \times \vec{B}$ (the Lorentz force, $\vec{J}$ being the electric current density and $\vec{B}$ being the magnetic induction vector) as the only body force are

$$\tau_{ij,j} + \left(\vec{J} \times \vec{B}\right)_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{2}$$

where $\left(\vec{J} \times \vec{B}\right)_i$ is the x_i-component of the force $\left(\vec{J} \times \vec{B}\right)$. Here, interaction of mechanical and electromagnetic fields is considered.

Let $u_i = (u_1, v_1, w_1)$ and denoting $x_1 = x,\ x_2 = y,\ x_3 = z$ then Eq. (2) can be written as

$$\left.\begin{aligned}\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_x &= \rho \frac{\partial^2 u_1}{\partial t^2}, \\ \frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_y &= \rho \frac{\partial^2 v_1}{\partial t^2}, \\ \frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_z &= \rho \frac{\partial^2 w_1}{\partial t^2}.\end{aligned}\right\} \tag{3}$$

For SH wave propagating in the x-direction and causing displacement in the y-direction only, we shall assume that

$$u_1 = w_1 = 0, \quad v_1 = v_1(x,z,t) \quad \text{and} \quad \frac{\partial}{\partial y} \equiv 0. \tag{4}$$

Using Eq. (4) in Eq. (3), we have

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_y = \rho \frac{\partial^2 v_1}{\partial t^2}, \tag{5}$$

where

$$\begin{aligned}\tau_{12} &= \mu_T \frac{\partial v_1}{\partial x} + (\mu_L - \mu_T)a_1\left(a_1 \frac{\partial v_1}{\partial x} + a_3 \frac{\partial v_1}{\partial z}\right), \\ \tau_{23} &= \mu_T \frac{\partial v_1}{\partial z} + (\mu_L - \mu_T)a_3\left(a_1 \frac{\partial v_1}{\partial x} + a_3 \frac{\partial v_1}{\partial z}\right).\end{aligned}$$

For stresses τ_{12} and τ_{23}, the first part indicate the shear stress due to elastic members (steel) and the second part indicates effect of comparatively non-elastic material of the composite section in the same direction.

The first component would be having the term μ_T, which can be termed as elastic coefficient and $(\mu_L - \mu_T)$ in the second term amounts for the effect comparatively non-elastic portion of the composite material.

The well-known Maxwell's equations governing the electromagnetic field are

$$\left.\begin{aligned}&\vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\tfrac{\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{H} = \vec{J}, \\ &\vec{B} = \mu_e \vec{H} \ \text{ and } \ \vec{J} = \sigma\left(\vec{E} + \tfrac{\partial u_i}{\partial t} \times \vec{B}\right),\end{aligned}\right\} \tag{6}$$

where $\vec{E}$ is the induced electric field, $\vec{J}$ is the current density vector and magnetic field $\vec{H}$ includes both primary and induced magnetic fields. μ_e and σ are the induced permeability and conduction coefficient, respectively.