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Magnetic field and magneto elastic stress in an infinite plate containing an elliptical hole with an edge crack under uniform electric current

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ABSTRACT

Two-dimensional solutions of the electric current, magnetic field and magneto elastic stress are presented for a magnetic material of a thin infinite plate containing an elliptical hole with an edge crack under uniform electric current. Using a rational mapping function, the each solution is obtained as a closed form. The linear constitutive equation is used for the magnetic field and the stress analyses. According to the electro-magneto theory, only Maxwell stress is caused as a body force in a plate which raises a plane stress state for a thin plate and the deformation of the plate thickness. Therefore the magneto elastic stress is analyzed using Maxwell stress. No further assumption of the plane stress state that the plate is thin is made for the stress analysis, though Maxwell stress components are expressed by nonlinear terms. The rigorous boundary condition expressed by Maxwell stress components is completely satisfied without any linear assumptions on the boundary. First, electric current, magnetic field and stress analyses for soft ferromagnetic material are carried out and then those analyses for paramagnetic and diamagnetic materials are carried out. It is stated that the stress components are expressed by the same expressions for those materials and the difference is only the magnitude of the permeability, though the magnetic fields H_x, H_y are different each other in the plates. If the analysis of magnetic field of paramagnetic material is easier than that of soft ferromagnetic material, the stress analysis may be carried out using the magnetic field for paramagnetic material to analyze the stress field, and the results may be applied for a soft ferromagnetic material. It is stated that the stress state for the magnetic field H_x, H_y is the same as the pure shear stress state. Solving the present magneto elastic stress problem, dislocation and rotation terms appear, which makes the present problem complicate. Solutions of the magneto elastic stress are nonlinear for the direction of electric current. Stresses in the direction of the plate thickness are caused and the solution is also obtained. Figures of the magnetic field and stress distribution are shown. Stress intensity factors are also derived and investigated for the crack length and the electric current direction.

1. Introduction

In materials, there are some defects like cracks and voids; therefore, the investigation of structural integrity and material deterioration are important to magnetic materials exposed to electromagnetic field. Also in modern technology, electric current operates in components such as power semiconductor devices, large scale integrated circuits, and huge electromagnetic systems. Therefore, many magnetic stress analyses have been carried out. Some reviews for magneto-solid mechanics were given by Paria (1967), Moon (1978, 1984), Pao (1978), Liang et al. (2002) and Fang et al. (2008). Hasebe et al. also gave some references in (2007, 2008, 2009a, 2009b, 2010a, 2010b). There are some models for the magneto stress analysis for soft ferromagnetic materials, i.e., Maxwell stress, pole, dipole, and Ampere current models (Moon, 1978, 1984). As a pioneer work, Pao and Yeh (1973) developed a linear theory for a soft ferromagnetic elastic solids based on the magnetic dipole model.

From a mathematical point of view, Maxwell's equations require a solution of certain boundary value problems, which are essentially three-dimensional ones for the magnetic field. Generally speaking, because three-dimensional boundary value problems are more difficult than those of two-dimensional one, many problems have been modeled and analyzed as the two-dimensionalized problems. Though one of the most important things is to obtain the magnetic field in the magnetic material of two-dimension, it seems not to be easy to make the two-dimensional model of the magnetic field. However, when the plate is thin, the magnetic field in the

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plate with a hole can be obtained. Therefore, the analysis of the magnetic field is carried out for the thin plate and also plane stress analysis can be applied, because the plate is thin.

According to the electro-magneto theory, only Maxwell stress components are caused as the body force in the magnetic material; therefore, Maxwell stress is considered for the stress analysis. No further assumptions for the magnetic stress analysis are made except the assumption of the plane stress state that the plate is thin, though Maxwell stress components and the boundary condition are expressed by the nonlinear terms of Maxwell stress components. The analysis is straightforward and the results of the stress seem to be acceptable. Electric current gives rise to electromagnetic field, and then causes electromagnetic force, Joule heat, temperature increase, heat flux and thermal stress. In the previous paper, analyses of electric current, Joule heat, temperature, heat flux and thermal stress caused by steady state electric current in a thin infinite plate containing an elliptical hole (Hasebe et al., 2009a) and with an edge crack (Hasebe et al., 2010a) and a strip with an edge notch or crack (Hasebe, 2010b) were reported. The magneto elastic stress caused by the magnetic field induced by electric current in a thin infinite plate with an elliptical hole is analyzed by Hasebe et al. (2009a).

In the present paper, magnetic field due to electricity and then the magneto elastic stress caused by steady state electric current in a thin infinite plate containing an elliptical hole with an edge crack are analyzed. Intensities of the magnetic field component and stress intensity factors at the crack tip are obtained.

Hasebe et al. analyzed the magnetic stress in an infinite plate with a square hole with an edge crack (2001), or an arbitrary hole (2007) subjected to the uniform magnetic field in the plate. The relationship between the magnetic stress analyses of Hasebe et al. (2001, 2007) and the present paper subjected to the magnetic field caused by the electric current is stated. One of the main reasons for the difficulty of solving the present problem is that it seems to achieve the two-dimensional magnetic field in the plate containing an elliptical hole with an edge crack under uniform electric current. Another issue may be that Maxwell stress components and the boundary conditions are expressed by nonlinear terms. In the present stress analysis, a dislocation and rotation terms appear, which also complicates the problem. The electric conductor is isotropic and homogenous in the plate. The plate is thin; therefore, it is assumed that the electric current density is uniform through the plate thickness. It is also assumed that the material constants do not depend on temperature.

Using a rational mapping function, closed form solutions are obtained for each problem of the electric current, magnetic field, and magnetic stress. To the best of our knowledge, though the present problem is one of the fundamental problems, it seems not to have been solved analytically.

2. Mapping function

The coordinate axes shown in Fig. 1 are denoted by x, y and z, respectively. The complex variable "w" is defined as $w = x + iy$ to

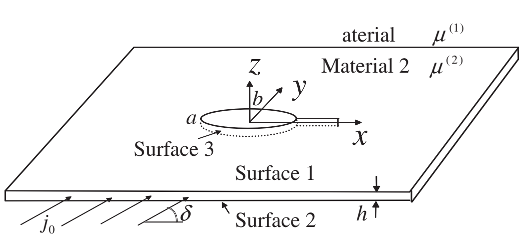

Fig. 1. Infinite thin plate containing an elliptical hole with an edge crack under uniform steady electric current with δ direction.

avoid confusion for "z" of the coordinates. The mapping function which maps the exterior of the elliptical hole containing an edge crack in the w-plane to the exterior of the unit circle in the ζ-plane shown in Fig. 2 is given by the following equation (Hasebe and Chen, 1996; Hasebe et al., 2010a):

$$w = x + iy = \omega(\zeta) = a\left[\frac{1+h_1}{2}\left(\zeta+\frac{1}{\zeta}\right)+h_1+\lambda\frac{1+h_1}{2}(\zeta+1)\left(1-\frac{\gamma_1}{\zeta}\right)^{1/2}\left(1-\frac{\gamma_2}{\zeta}\right)^{1/2}\right] \tag{1}$$

where

$$e_1 = c/a, \quad \lambda = b/a$$
$$h_1 = \frac{e_1^2}{4(1+e_1)} \quad (\text{for } \lambda = 1, \text{circular hole})$$
$$h_1 = \frac{(e_1+\lambda^2) - \lambda\sqrt{\lambda^2+2e_1+e_1^2}}{2(1-\lambda^2)} \quad (\text{for } \lambda \neq 1, \text{elliptical hole})$$
$$\left.\begin{matrix}\gamma_1\\ \gamma_2\end{matrix}\right\} = \frac{1-h_1}{1+h_1} \pm i\frac{2\sqrt{h_1}}{1+h_1} \tag{2a—e}$$

where a and b are the semi-axes of the elliptical hole, and c is the crack length. When a mapping function is a rational one, closed form stress functions are obtained (Muskhelishvili, 1963). Because (1) is not a rational function, a rational mapping function is formed as a sum of fractional expressions to facilitate the basic requirement for a closed form solution. The rational mapping function is formed as

$$w = \omega(\zeta) = F_0\zeta + \sum_{k=1}^{n}\frac{F_k}{\zeta_k-\zeta} + \frac{F_{25}}{\zeta} + F_c \tag{3}$$

where $n = 24$ is used in this paper, and F_0, F_k ($k = 1,2,\ldots,25$) and F_c are constants. Poles ζ_k ($k = 1,2,\ldots,n$) are located inside the unit circle. The formulation of this rational mapping function was stated in (Hasebe and Horiuchi, 1978; Hasebe and Wang, 2005). When coefficients $F_k = 0$ ($k = 1,2,\ldots,n$), $F_0 = (a+b)/2$ and $F_5 = (a-b)/2$, the hole becomes an elliptical one where 'a' and 'b' are semi-axes of the elliptical hole. And the hole is a circle for $a = b(\lambda = 1)$, and a crack for $b = 0$ ($\lambda = 0$). The magnitude of a radius, ρ, of curvature at the crack tip of (3) is $\rho/a = 10^{-9} - 10^{-11}$ which depends on the crack length, and is very small. The radii of curvature at convex points K and H are also small, reaching zero for irrational mapping function (1).

One of the main merits using a rational mapping function is that stress functions achieved are exact ones for the geometrical shape represented by the rational mapping function. A rational mapping function of a sum of fraction expressions is also applied to any complicated configuration in principle (Hasebe and Horiuchi,

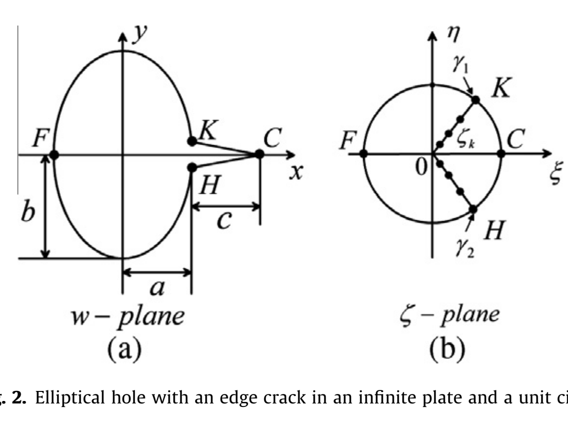

Fig. 2. Elliptical hole with an edge crack in an infinite plate and a unit circle.