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Bistable plates for morphing structures: A refined analytical approach with high-order polynomials

A. Pirrera [a,*,1], D. Avitabile [b,**,1], P.M. Weaver [a]

[a] Aerospace Engineering Department, University of Bristol, Queen's Building, University Walk, Bristol, BS8 1TR, UK
[b] Engineering Mathematics Department, University of Bristol, Queen's Building, University Walk, Bristol, BS8 1TR, UK

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ABSTRACT

The multistability of composite thin structures has shown potential for morphing applications. The present work combines a Ritz model with path-following algorithms to study bistable plates' behaviour. Classic low-order Ritz models predict stable shapes' geometry with reasonable accuracy. However, they may fail when modelling other aspects of the elastic structural behaviour. A refined higher-order model is here presented. In order to improve the inherently poor conditioning properties of Ritz approximations of slender structures, a non-dimensional version of Classical Plate Lamination Theory with von Kármán nonlinear strains is developed and presented. In the current approach, we continue numerical solutions in parameter space, that is, we path-follow equilibrium configurations as the control parameter varies, find stable and unstable configurations and identify bifurcations. The numerics are carried out using a set of in-house MATLAB® routines for numerical continuation. The increased degrees of freedom within the model are shown to accurately reflect buckling loads and provide useful insight into the relative importance of different aspects of nonlinear behaviour. Finally, the complex, experimentally observed snap-through geometry is captured analytically for the first time. Results are validated against finite elements analysis throughout the course of the paper.

1. Introduction

The present paper introduces a systematic approach for the study of a distinct nonlinear structural problem i.e. multistability of composite panels.

In the recent past, the subject has received significant attention because multistable slender panels have shown potential for morphing applications (e.g. Dano and Hyer, 1998; Seffen, 2007; Kebadze et al., 2004).

Nevertheless, since dealing with their complex nonlinear behaviour has proven to be a challenging task, current analysis and design methodologies suffer from practical limitations and, most importantly, do not capture some of the details which are crucial for potential engineering applications (Schlecht and Schulte, 1999; Gigliotti et al., 2004; Cerami and Weaver, 2008; Diaconu et al., 2009). A better understanding is, thus, key for a full exploitation of multistable panels as morphing technology.

The approach proposed here has been conceived to overcome current modelling limitations and to provide the means for a more efficient design. Since multistable panels feature very large displacements and large rotations, this has been done starting from questioning fundamental assumptions at the very base of current analysis approaches. The effectiveness of a Lagrangian description of the structural behaviour by means of the Classic Plate Lamination Theory with von Kármán nonlinearities is indeed not to be taken for granted. In addition, in order to investigate accurately and systematically multistable panels' design space, numerical analyses are performed by combining high-order non-dimensional Ritz approximations and numerical continuation procedures. As a result, a tool for optimal design is obtained.

As an example of the utility of the method, a case study is carried out on the thermally-induced bistability of fibre-reinforced unsymmetric composite plates. It will be shown that it is possible to explore bistable plates' design space in a robust and systematic fashion and that high-order approximations capture in details their complex structural behaviour. Moreover, since this case study has frequently been explored, it is possible to compare the results presented against those available in literature and show that the level of accuracy that has been attained here is, in fact, fundamental for engineering applications. In particular, it is proven that high-order polynomial approximations capture:

* Corresponding author.
** Corresponding author.
E-mail addresses: alberto.pirrera@bristol.ac.uk (A. Pirrera), d.avitabile@bristol.ac.uk (D. Avitabile).

[1] A. Pirrera and D. Avitabile contributed equally.

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- The shape of the equilibrium configurations and hence that a Lagrangian description of the equilibrium with von Kármán nonlinearities is appropriate (for the lay-up used here) in spite of the very large displacements.
- The experimentally observed (Potter et al., 2007), multi-event snap-through event for the first time.
- The snap-through load with adequate approximation.
- The loss of bifurcation with plan-form geometry and aspect ratio, thus explaining Gigliotti et al.'s observations (Gigliotti et al. (2004)).

To conclude this introduction, a concise review of the literature covering the subject is given. In the following sections the development of the proposed modelling technique is outlined.

1.1. Literature review

In a similar way to bimetallic strips, unsymmetric composite laminates convert temperature changes in displacements. When cured flat, unsymmetric laminates may warp and assume two stable cylindrical configurations at room temperature. The existence of another saddle-like unstable configuration is predicted theoretically. The physics behind this phenomenon is easily explained by the mismatch of the thermal expansion coefficients of the layers that compose the whole structure. In the mono-dimensional strip, the metals tend to expand with different rates; to accommodate this differential the strip bends laterally. The two-dimensional equivalent has one more feature due to the directionality of the expansion. Indeed, the orthotropic layers expand with different rates in different directions. This characteristic may result in the instability of the highly-stretched saddle deformation mode and to the existence of two high displacements bending modes which allow the laminate to accommodate the differential expansion with a lower strain energy content.

Bimetallic strips were probably invented by eighteen-century clockmaker John Harrison (Sobel, 1995). Nowadays, they are still widely used as components of thermostats, thermometers and circuit breakers. The history of bistable composite plates begins in the more recent past. In the 1980's unsymmetric laminates were used to test environment dependent material systems' properties. In 1981 Hyer presented a study on the properties of many families of cross-ply laminates. Successively, in order to describe bistability, Hamamoto and Hyer (1987) incorporated geometric von Kármán nonlinearities in the Classic Plate Lamination Theory (CLPT). The system of equations thereby generated was then solved with a Ritz minimisation of the total potential energy.

In fact, Hyer generated a lot of interest and over the following years many researcher contributed to refine its model (Dang and Tang, 1986; Jun and Hong, 1990; Hamamoto and Hyer, 1987; Jun et al., 1992; Peeters et al., 1996; Dang and Tang, 1986; Schlecht et al., 1995). In the late 1990's, as the concept of structural morphing started to be mainstream between researchers, interest reinvigorated. In 1996 Dano and Hyer included the contribution of applied forces into the Ritz minimisation of the total potential energy. In 1998 they introduced a new model, hitherto considered state-of-the-art, in which they assumed a possible strain rather than displacement field. Schlecht and Schulte (1999) presented an extensive Finite Element (FE) study pointing out the existence of a complex local deformation behaviour that was not captured by many of the analytical models proposed at that time.

A number of papers dealing with the possibility of actuating bistable plate with smart material were published as the snap-through phenomenon became central (Hyer et al., 1998; Dano and Hyer, 2002; Dano and Hyer, 2003; Schultz et al., 2003; Schultz and Hyer, 2004; Schultz, 2005; Portela et al., 2005; Portela et al., 2008; Gude et al., 2006; Hufenbach et al., 2006; Ren, 2007; Bowen et al., 2007; Potter et al., 2007; Giddings et al., 2008; Vidoli and Maurini, 2008; Fernandes et al., 2010). Manufacturing aspects were also studied (Cho et al., 2003; Gigliotti et al., 2003; Gigliotti et al., 2006) and parametric analyses were performed to investigate the domain of existence of bistability (Hufenbach et al., 2001; Hufenbach et al., 2002; Hufenbach et al., 2002; Gigliotti et al., 2004). Eventually, new concepts for morphing applications were presented (Potter and Weaver, 2004; Schultz, 2005; Schultz, 2008; Mattioni et al., 2006; Mattioni et al., 2008; Diaconu et al., 2008; Diaconu et al., 2009; Daynes, 2009) and, in an attempt to capture as much as possible of bistable systems' physics, modified modelling techniques were proposed (Ren, 2007; Ren and Mar, 2008; Cerami and Weaver, 2008; Carrella et al., 2008; Arrieta et al., 2009; Diaconu et al., 2009).

2. The modelling framework and its architecture

Bistable structures are typically characterised by the very large displacements and nonlinear deformations that are undertaken during structural shape changes. FE and analytical studies have already shown that these structures may have multiple stable and unstable equilibria (Hyer, 1981; Hamamoto and Hyer, 1987; Dano and Hyer, 1996; Dano and Hyer, 1998; Schlecht and Schulte, 1999). The current aim is to investigate the design space for bistable composite plates in a systematic and robust fashion. In principle, it is desirable to understand how such structures behave when one or more design parameters are changed. A further aim is to assess whether our model, which uses a Lagrangian description of equilibria with von Kármán strains is adequate to capture the very large displacements (and small strains) or whether an Eulerian description of equilibria (shape dependent equilibria) coupled with more accurate strain descriptors would be necessary. Many researchers have contributed over the years, e.g. Pietraszkiewicz (1984).

For design purposes, parametric studies are usually carried out running simulations with commercial Finite Element software or by using analytical approaches based on standard discretisation methods. So far, analytical procedures have been shown to be more suitable. The reasons are as follows.

Firstly, geometrical properties, such as the side length, appear explicitly in the equilibrium equations. It is, therefore, appropriate to treat them as parameters and, as a consequence, it is easier to analyse systematically a wider design space.

Secondly, when based on modal decomposition methods such as Ritz or Galerkin, parametric analyses are less computationally demanding; the investigation of the design space is thus more feasible, both in terms of hardware and time. The main advantage of these decomposition methods is that they produce accurate results with a relatively low number of unknowns. This is because the semi-analytical approach relies on a suitable modal basis which can be chosen to give the maximum amount of information with the minimum number of unknowns. In other words, it can give good insight with a relatively low number of "specialised" degrees of freedom and with little need for computational power. For instance, the modal decomposition can reflect the symmetries of the original structure, and this can be used to rule out unwanted modes. Conversely, discretisation methods based on a mesh definition, such as Finite Elements, Spectral Methods or Finite Differences, tend to give good results only with a large number of nodes i.e. "generic degrees of freedom".

In the present study, in line with the vast majority of previous works on bistability-related problems, the Ritz method is used. As a novelty, the latter methodology is complemented by numerical continuation routines based on path-following algorithms. Regardless of the discretisation method, this approach gives the means for a systematic and parametric exploration of the design