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Analytical investigation for true and spurious eigensolutions of multiply-connected membranes containing elliptical boundaries using the dual BIEM

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ABSTRACT

It is well known that the boundary element method may induce spurious eigenvalues while solving eigenvalue problems. The finding that spurious eigenvalues depend on the geometry of inner boundary and the approach utilized has been revealed analytically and numerically in the literature. However, all the related efforts were focused on eigenproblems involving circular boundaries. On the other hand, the extension to elliptical boundaries seems not straightforward and lacks of attention. Accordingly, this paper performs an analytical investigation of spurious eigenvalues for a confocal elliptical membrane using boundary integral equation methods (BIEM) in conjunction with separable kernels and eigenfunction expansion. To analytically study this eigenproblem, the elliptic coordinates and Mathieu functions are adopted. The fundamental solution is expanded into the separable kernel by using the elliptic coordinates and the boundary densities are expanded by using the eigenfunction expansion. The Jacobian terms may exist in the separable kernel, boundary density and boundary contour integration and they can cancel each other out. Therefore, the orthogonal relations are reserved in the boundary contour integration. In this way, a similar finding about the mechanism of spurious eigenvalues is found and agrees with those corresponding to the annular case. To verify this finding, the boundary element method and the commercial finite-element code ABAQUS are also utilized to provide eigensolutions, respectively, for comparisons. Good agreement is observed from comparisons. Based on the adaptive observer system, the present approach can deal with eigenproblems containing circular and elliptical boundaries at the same time in a semi-analytical manner. By using the BIEM, it is found that spurious eigenvalues are the zeros of the modified Mathieu functions which depend on the inner elliptical boundary and the integral formulation. Finally, several methods including the CHIEF method, the SVD updating technique and the Burton & Miller method are employed to filter out the spurious eigenvalues, respectively. In addition, the efficiency of the CHIEF method is better than those of the SVD updating technique and the Burton & Miller approach, since not only hypersingularity is avoided but also computation effort is saved.

1. Introduction

Eigenanalysis is very important for vibration and acoustics, because it can provide some fundamental information. Since analytical solutions are sometimes not available, numerical methods are needed. In recent years, several numerical methods were utilized to determine eigenvalues and eigenmodes such as the finite element method (FEM) or the boundary element method (BEM). Although the FEM is a popular method, it needs to generate the mesh over the whole domain. The BEM only generates the mesh on the boundary but it may face with the calculation of the principal value and the pollution of spurious eigenvalues. Tai and Shaw (1974) first employed the complex-valued BEM to solve membrane vibration. De Mey (1976) revisited this problem in 1976. Later, De Mey (1977) proposed a simplified approach by using only the real-part or imaginary-part kernel where he found that spurious solutions were imbedded as well as the ill-posed matrix appeared. In a similar way of using the real-part kernel, Hutchinson and Wong (1979) and Hutchinson (1984) solved the free vibration of plate. Also, Yasko (2000) as well as Duran et al. (2001) employed the real-part kernel approach. It is interesting to find that Kang et al. (1999) proposed a non-dimensional influence function (NDIF) method which was an imaginary-part kernel approach as

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commented by Chen et al. (2002). Kamiya et al. (1996) and Yeih et al. (1998) linked the relation of the multiple reciprocity method (MRM) and real-part BEM independently. This is the reason why spurious eigenvalues are also inherent in these two methods, the MRM and the real-part BEM. However, no proof was given at that time. Until 2000, Kuo et al. (2000) proved the existence of spurious eigensolutions and pointed out that spurious eigenvalues occur at the zeros of the mth-order Bessel functions of the second kind or their derivatives through a circular membrane for the real-part dual BEM. Later, Chen et al. (2004a), Lee and Chen (2008a) extended a circular membrane to a circular plate by using the real-part BEM and BIEM, respectively.

Appearance of spurious eigensolutions for simply-connected problems was due to the loss of constraint by only using the real or the imaginary-part kernel (Kuo et al., 2000; Chen et al., 2009a). Tai and Shaw (1974) claimed that spurious eigenvalues do not appear if the complex-valued kernels are employed. However, it is true only for simply-connected domain. Even though we employ the complex-valued kernels for the multiply-connected eigenproblems, the spurious eigensolutions also occur (Kitahara, 1985; Chen et al., 2001, 2003). Kobayashi and Nishimura (1982) have identified that spurious eigenvalues (irregular frequencies) for multiply-connected domains using the singular formulation are the eigenfrequencies of the Dirichlet problem containing the inner boundary. From their arguments, it is quite obvious that the spurious eigenvalues in the singular (resp. hypersingular) formulation for multiply-connected domains are the Dirichlet (resp. Neumann) eigenfrequencies of problems containing the inner boundary. Since the doubly-connected eigenproblem can be decomposed into two parts, one is an exterior problem bounded by interior boundary and the other is an interior problem. Mathematically speaking, a spurious eigenvalue for the doubly-connected problem originates from the same fictitious frequency for the exterior part problem. It is interesting to find that the spurious eigenvalues of the annular domain depend on the geometry of inner boundary and the integral formulation. This finding was analytically verified by Chen et al. (2003). This finding is the same with the viewpoint of Kobayashi and Nishimura (1982).

It is well known that fictitious frequency appears in the exterior Helmholtz problem. The Burton & Miller approach (Burton and Miller, 1971) and the combined Helmholtz interior integral equation formulation (CHIEF) method (Schenck, 1968; Seybert and Rengarajan, 1987) have been proposed to deal with the fictitious frequency problem of the exterior acoustics. Furthermore, Chen et al. (2001, 2003, 2007) extended the Burton & Miller approach as well as the CHIEF method to filter out the spurious eigenvalues for the multiply-connected eigenproblems. Spurious eigenvalues can be fully filtered out by using the Burton & Miller approach, but it needs the computation of the hypersingular equation. The CHIEF method may fail if one uses inappropriate CHIEF points in order to add independent equations. If the location of the CHIEF point is located on the nodal line of the interior problems (Chen et al., 2003), the appearance of fictitious frequency may not be suppressed. Besides, Chen and his co-workers (2003, 2004b, 2005, 2007, 2008b, 2009a, 2010a) employed the singular value decomposition (SVD) updating technique to detect spurious eigenvalues. Not only true eigenvalues but also spurious eigenvalues can be extracted out by using the SVD updating terms and SVD updating documents, respectively. This technique has been successfully employed for rod (Chen et al., 2009a), circular membrane (Chen et al., 2003, 2005, 2007), plate (Lee and Chen, 2008b) and concentric sphere cavity (Chen et al., 2010a).

Chen et al. (2001, 2003) studied spurious eigenvalues of circular membranes in both the continuous and discrete systems by using the separable kernel and circulant, respectively. Note that the term of the separable kernel is adopted in the paper to imply an infinite-rank expansion of a closed-form fundamental solution. It is to avoid the confusion with the degenerate kernel defined by Courant and Hilbert (1989) to be a kernel with a finite-rank expansion to approximate the closed-form fundamental solution. In numerical implementation, we adopt the separate kernel for the fundamental solution since finite terms of expansion are considered. In the paper, we aim to extend successful experiences in annular membranes (Chen et al., 2001, 2003) to deal with eigenproblems containing elliptical boundaries. However, the circulant property is no longer present for the ellipse. A special care should be taken to derive a separable kernel in the elliptic coordinates which is not straightforward to obtain. Regarding eigenproblems with elliptical boundaries, Troesch and Troesch (1973) used the separation of variables to obtain the eigenfrequencies and nodal patterns of an elliptic membrane. Hong and Kim (1995) also employed the separation of variables to determine the natural mode of hollow and elliptical annulus for cylindrical cavities. Both the elliptic coordinates and the Mathieu functions were used in the previous investigations.

Recently, Chen et al. (2007) developed the null-field boundary integral equation method (BIEM) in conjunction with the separable kernel and the Fourier series to solve the eigenproblems containing circular boundaries. By introducing the separable kernels, the calculation of the singular and hypersingular integrations in the sense of principal value by using bump contour are free when the collocation point is exactly located on the real boundary. This approach is one kind of semi-analytical and meshless methods. Later, Chen et al. (2010b) extend the BIEM to deal with torsion problems containing multiple elliptical inclusions. The key point is that the separable kernel in terms of the elliptic coordinates is available in the Morse and Feshbach's book (1953).

In this paper, we will derive the BIE formulations and extend to solve eigenproblems containing elliptical boundaries. The BIEM is utilized in conjunction with the separable kernel and the eigenfunction expansion for the closed-form fundamental solution and boundary densities, respectively. To fully utilize the elliptical geometry, the elliptic coordinates in companion with the Mathieu function (Morse and Feshbach, 1953; Abramowitz and Stegun, 1965; Zhang and Jin, 1996) are used. The fundamental solution is expanded to the separable kernel by using the elliptic coordinates (Morse and Feshbach, 1953). Also, the boundary densities are expanded by using the eigenfunction expansion in companion with a Jacobian term. The advantage of free of calculating principal value is gained. Following the successful experience of the annular case (Chen et al., 2003), we extend to the elliptical case in this paper. In order to analytically verify the occurring mechanism of the spurious eigenvalues for multiply-connected problem containing elliptical boundaries, the confocal elliptical membrane is considered. The boundary element method is also implemented to demonstrate the finding by using the present approach. Furthermore, the commercial finite-element code ABAQUS is also utilized to provide eigensolutions for comparisons. As mentioned above, it is already known in the literature by Kobayashi and Nishimura (1982) that spurious eigensolutions depend on the geometry of inner boundary and the approach used. However, all the related efforts were focused on eigenproblems involving circular boundaries. Spurious eigenvalue of a confocal elliptical membrane have not been analytically studied in detail by using the BIEM to the authors' best knowledge. Accordingly, this paper is focused on analytically deriving the true and spurious eigenequations of a confocal elliptical membrane by using the BIEM. Finally, we also employ CHIEF method, the SVD updating technique and the Burton & Miller approach to suppress the spurious appearance of eigenvalues.