International Journal of Solids and Structures 47 (2010) 1–9

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Dihedral 'star' tensegrity structures

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ARTICLE INFO

Article history:
Received 26 September 2008
Received in revised form 10 May 2009
Available online 13 June 2009

Keywords:
Tensegrity structure
Dihedral symmetry
Block-diagonalisation
Self-equilibrium
Stability

ABSTRACT

This paper presents conditions for self-equilibrium and super stability of dihedral 'star' tensegrity structures, based on their dihedral symmetry. It is demonstrated that the structures are super stable if and only if they have an odd number of struts, and the struts are as close as possible to each other. Numerical investigations show that their prestress stability is sensitive to the geometry realisation.

1. Introduction

In this paper, we describe the equilibrium and stability of dihedral 'star' tensegrity structures which we derive from the classic dihedral prismatic tensegrity structures. Both of these two classes of structures are of dihedral symmetry, and therefore, the stability properties of the star structures can be investigated by the methods for the prismatic structures in our early studies (Zhang et al., 2009a,b).

The horizontal cables in each of the two parallel circles containing the nodes in a prismatic structure are replaced by a star of cables in a 'star' structure, with a new centre node. An example 'star' structure is shown in Fig. 1(b), along with the parent prismatic structure in Fig. 1(a). Also shown in Fig. 1(c) is a modified version of the structure, where there exists a centre member connected to the two centre nodes.

There is a clear link between the star structures, and the parent prismatic structures that were studied by Connelly and Terrell (1995) and Zhang et al. (2009a). Indeed, we shall see that the equilibrium positions of the nodes, and self-stress forces in the vertical cables and the struts, are identical in the star and prismatic structures, as long as there is no centre member. However, the star structure has many more infinitesimal mechanisms than its parent prismatic structure: at each of the boundary nodes, a strut is in equilibrium with two cables, all of which must therefore lie in a plane; thus, out-of-plane movement of the node must be an infinitesimal mechanism, and there are at least six infinitesimal mechanisms – in fact there is another infinitesimal mechanism corresponding to the existence of one self-stress mode of the structure. By contrast, there is only one infinitesimal mechanism in the prismatic tensegrity structure. Despite this, we will show that many dihedral star tensegrity structures can be stable, and further, that in some cases they are *super* stable, which implies that they are stable for any level of self-stress, independently of the stiffness of the members.

Following this introduction, the paper is organized as follows: Section 2 uses the symmetry of a star structure to find its configuration and self-stress forces in the state of self-equilibrium. Section 3 presents the necessary and sufficient condition for an 'indivisible' structure. Section 4 block-diagonalises the force density matrix and finds the condition, in terms of connectivity of vertical cables, for super stability of the star structures; prestress stability of the structures that are not super stable is numerically investigated. Section 5 briefly concludes the study on the star structures, and discusses the stability properties of those with centre members.

2. Configuration

In this section, we introduce the connectivity and geometry of a general star structure, and find the internal forces that equilibrate every node. The structure has dihedral symmetry, and this symmetry allows us to calculate symmetric state of self-stress by considering the equilibrium equations of only representative nodes.

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0020-7683/$ - see front matter © 2009 Published by Elsevier Ltd.
doi:10.1016/j.ijsolstr.2009.05.018

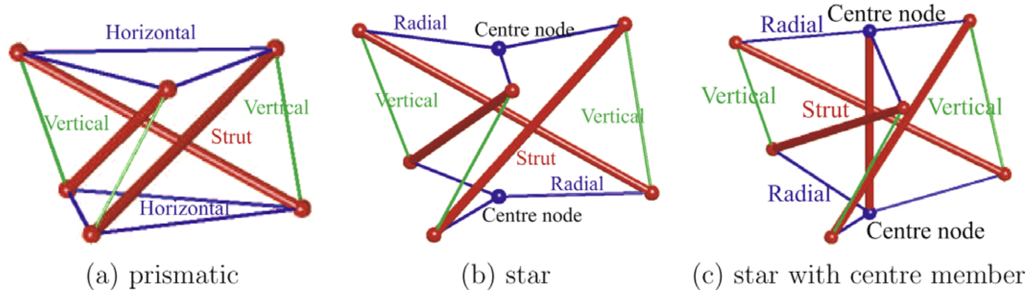

Fig. 1. Tensegrity structures that are of the same dihedral symmetry $\mathbf{D}_3$. The thick lines represent struts, and the thin lines represent cables.

2.1. Symmetry and connectivity

We are considering star tensegrity structures that have dihedral symmetry, denoted by $\mathbf{D}_n$ (in the Schoenflies notation, see for example Kettle (1995)): there is a single major n-fold rotation (C_n^i) axis, which we assume is the vertical, z-axis, and n twofold rotation ($C_{2,j}$) axes perpendicular to this major axis. In total there are $2n$ symmetry operations. A star structure has the same appearance before and after the transformation by applying any of these symmetry operations.

Consider a specific set of elements (nodes or members) of a structure with symmetry G. If one element in a set can be transformed to all of the other elements of that set by the symmetry operations in G, then this set of elements are said to belong to the same *orbit*. A structure can have several different orbits of elements.

In contrast to prismatic structures, which have only one orbit of nodes, there are two orbits of nodes in star structures – boundary nodes and centre nodes, as shown in Fig. 2:

- There are $2n$ 'boundary' nodes arranged in two horizontal circles of radius R around the vertical z-axis; there is a one-to-one correspondence between the boundary nodes and the symmetry operations. (When there is a one-to-one correspondence between elements and symmetry operations, the orbit is called a *regular orbit*.)
- There are two 'centre' nodes that lie on the centres of the two horizontal circles; the cyclic (n-fold) rotation operations do not change the locations of these nodes, while the twofold rotation operations swap their positions.

Thus, there are in total $2n+2$ nodes. The two horizontal circles containing the boundary nodes are at $z = \pm H/2$, and the centre nodes are also at $z = \pm H/2$, as shown in Fig. 2(c).

There are three orbits of members: radial cables, vertical cables and struts. The members in each orbit have the same length and internal force, and therefore, the same force density (ratio of internal force to length). Each of the boundary nodes in a circle is connected by a 'radial' cable to a centre node. Hence, there are $2n$ radial cables, and each symmetry operation transforms a radial cable into one of the other radial cables; i.e., there is a one-to-one correspondence between the radial cables and the symmetry operations (the radial cables form a regular orbit). Each boundary node is connected by a strut and a 'vertical' cable to boundary nodes in the other circle. Thus, there are only n vertical cables, and n struts: there is a one-to-two correspondence between the vertical cables (or struts) and the symmetry operations. Each vertical cable and strut intersects one of the twofold horizontal rotation axes, and this twofold operation transforms this vertical cable (or strut) into itself.

It is possible to have different connectivities of the vertical cables and struts for any $n > 3$. We use the notation $\mathbf{D}_n^v$ to describe the connectivity of a star tensegrity with $\mathbf{D}_n$ symmetry, where v describes the connectivity of the vertical cables, assuming that connectivity of struts is fixed. The boundary nodes in the upper and lower circles are, respectively, numbered as $N_0, N_1 \ldots, N_{n-1}$ and $N_n, N_{n+1} \ldots, N_{2n-1}$, and the upper and lower centre nodes are numbered as N_{2n} and N_{2n+1}, respectively. We describe the connectivity of a reference node N_0 as follows – all other connections are then defined by the symmetry.

(1) Without loss of generality, we assume that a strut connects node N_0 in the upper circle to node N_n in the lower circle.
(2) A radial cable in the upper circle connects node N_0 to the centre node N_{2n}, and a radial cable in the lower circle connects node N_n to the centre node $N_{2n} + 1$.
(3) A vertical cable connects node N_0 in the upper circle to node N_{n+v} in the lower circle. We restrict $1 < v < n/2$ (choosing $n/2 < v < n$ would give essentially the same set of structures, but in a reflection symmetry with respect to the plane $z = 0$).

The numbering of nodes of two example structures with $\mathbf{D}_5$ symmetry, $\mathbf{D}_5^1$ and $\mathbf{D}_5^2$, is shown in Fig. 3. Node N_0 is connected by a strut to node N_5, and by a vertical cable to node N_6 for $\mathbf{D}_5^1$, and to node N_7 for $\mathbf{D}_5^2$.

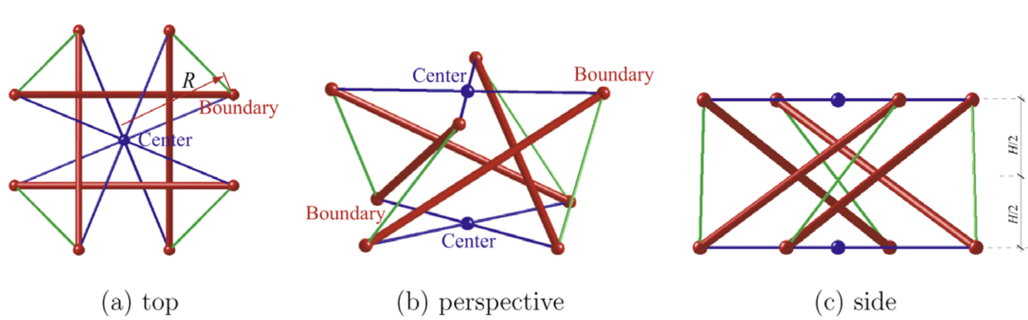

Fig. 2. The dihedral star tensegrity structure $\mathbf{D}_4^1$. R and H are the radius of the circle of boundary nodes and height of the structure, respectively.