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Analysis of cracked magnetoelectroelastic composites under time-harmonic loading

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ABSTRACT

This paper presents a numerical model for the analysis of cracked magnetoelectroelastic materials subjected to in-plane mechanical, electric and magnetic dynamic time-harmonic loading. A traction boundary integral equation formulation is applied to solve the problem in combination with recently obtained time-harmonic Green's functions (Rojas-Diaz et al., 2008). The hypersingular boundary integral equations appearing in the formulation are first regularized via a simple change of variables that permits to isolate the singularities. Relevant fracture parameters, namely stress intensity factors, electric displacement intensity factor and magnetic induction intensity factor are directly evaluated as functions of the computed nodal opening displacements and the electric and magnetic potentials jumps across the crack faces. The method is checked by comparing numerical results against existing solutions for piezoelectric solids. Finally, numerical results for scattering of plane waves in a magnetoelectroelastic material by different crack configurations are presented for the first time. The obtained results are analyzed to evaluate the dependence of the fracture parameters on the coupled magnetoelectromechanical load, the crack geometry and the characteristics of the incident wave motion.

1. Introduction

Magnetoelectroelastic composites that combine piezoelectric and piezomagnetic phases have drawn significant attention in recent years due their ability to convert energy among magnetic, electric and mechanical fields. This is the reason why they are finding increasing use in smart structures applications. Such magnetoelectromechanical coupling may be found as well in some single-phase materials where simultaneous magnetic and electric ordering coexist. However, the main advantage of such composites as compared to the single-phase materials is that the resulting electromagnetic coupling may be even a hundred times larger (Van Suchtelen, 1972; Nan, 1994; Eerenstein et al., 2006).

As the use of magnetoelectroelastic composites in engineering increases, fracture mechanics of this class of materials has become an emerging research front where a large effort is focused on understanding the failure mechanisms involved and, subsequently, on generalizing some of the tools previously developed for anisotropic or piezoelectric fracture analysis to the magnetoelectroelastic case. Significant contributions have been done by several authors for static fracture: works by Wang and Mai (2003, 2004, 2007), Gao et al. (2003a,b, 2004), Tian and Gabbert (2004, 2005) or Li and Kardomateas (2006) should be cited among others.

However, the analysis of dynamic fracture problems of magnetoelectroelastic materials is more limited so far, not to mention that the majority of such analysis deals with anti-plane fracture, using semi-analytical solution methods. Zhou and Wang used the Schmidt method to investigate the dynamic behavior of an interface crack in a magnetoelectroelastic composite under harmonic elastic anti-plane shear waves (Zhou and Wang, 2005), and further extended this technique to analyze the cases of two collinear symmetric interface cracks between two dissimilar magnetoelectroelastic half planes (Zhou et al., 2005), two parallel symmetry cracks (Zhou and Wang, 2006), two parallel symmetry interface cracks (Zhou et al., 2006), two collinear interface cracks between two dissimilar functionally graded piezoelectric/piezomagnetic material strips (Zhang et al., 2007) and an interface crack between two dissimilar functionally graded piezoelectric/piezomagnetic material half infinite planes subjected to harmonic anti-plane shear stress waves (Zhou and Wang, 2008). Hu and Li (2005) derived the analytical solution for an anti-plane Griffith moving crack inside an infinite magnetoelectroelastic medium under the assumption of permeable crack faces and later extended this study to the case of an anti-plane Griffith crack moving at the interface between two dissimilar magnetoelectroelastic media (Hu et al., 2006). Li (2005) investigated the transient response of a magnetoelectroelastic medium containing a crack along the poling direction subjected to antiplane mechanical and inplane electric and magnetic impacts. Feng and Su (2006)) analyzed the dynamic anti-plane problem for a functionally graded magnetoelectroelastic

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strip containing an internal crack perpendicular to the boundary, under both magnetoelectrically impermeable or permeable boundary conditions on the crack faces. Zhong and Li (2006) studied the dynamic problem of an anti-plane shear crack of finite length moving with a constant velocity along the interface of two dissimilar magnetoelectroelastic materials. Feng et al. (2007) analyzed the dynamic behavior induced by a penny-shaped crack in a magneto-electroelastic layer subjected to prescribed stress or prescribed displacement at the layer surfaces for both impermeable and permeable cracks. Su et al. (2007) studied the problem of an arbitrary number of interface cracks between dissimilar magnetoelectroelastic strips under out-of-plane mechanical and in-plane magneto-electrical impacts. Yong and Zhou (2007) considered the transient anti-plane problem of a magnetoelectroelastic strip containing a crack vertical to the boundary. Liang (2008) derived the solution for the dynamic behavior of two parallel symmetric cracks in functionally graded piezoelectric/piezomagnetic materials subjected to harmonic antiplane shear waves. Feng and Pan (2008) investigated the anti-plane problem for an interfacial crack between two dissimilar magnetoelectroelastic plates subjected to anti-plane mechanical and in-plane magneto-electrical impact loadings under different combinations of magnetically and electrically permeable/impermeable surface conditions on the crack. More recently, Sladek et al. (2008) presented a meshless method based on the local Petrov-Galerkin approach for stationary and transient dynamic crack analysis in two-dimensional and three-dimensional axisymmetric magnetoelectroelastic solids with continuously varying material properties.

In this paper, a dual BEM formulation is presented for the analysis of cracked fully anisotropic magnetoelectroelastic composite solids under in-plane mechanical, electric and magnetic dynamic time-harmonic loading. The approach presented herein is a generalization to magnetoelectroelastic materials of our previous works for dynamic fracture of anisotropic and piezoelectric media (García-Sánchez et al., 2006; Sáez et al., 2006). The time-harmonic fundamental solution derived by Rojas-Diaz et al. (2008) using the Radon transform is split into singular plus regular parts. The singular part is independent of the frequency and it coincides with the static fundamental solution. In this way, the treatment of all the singularities arising in the BEM formulation follows the same regularization scheme presented by García-Sánchez et al. (2007) for statics, so that in order to solve the dynamic problem only regular terms need to be added to the static BEM. Quadratic elements are introduced for the boundary discretization, with straight quarter-point elements at the crack tips to adequately model the behavior of the field variables around the crack tips. Stress intensity, electric displacement and magnetic induction intensity factors are directly extrapolated from the computed nodal values at the quarter-point elements. The formulation is validated by considering wave scattering by a Griffith crack in a degenerate quasi-piezoelectric solid, since for this problem analytical solutions are available (Shindo and Ozawa, 1990). Some other numerical results are presented for a $BaTiO_3$–$CoFe_2O_4$ composite material to illustrate the accuracy and possibilities of the present approach.

2. Numerical modeling of dynamic fracture by BEM

The behavior of stationary cracks subjected to dynamic time-harmonic loading in plane magnetoelectroelastic solids will be addressed by a dual BEM formulation (Hong and Chen, 1988; Portela et al., 1992). The approach summarized within this section is an extension to the magnetoelectroelastic case of our previous work for anisotropic and piezoelectric crack problems (García-Sánchez et al., 2006; Sáez et al., 2006).

2.1. Constitutive equations

Consider a homogeneous, linear and fully anisotropic magneto-electroelastic plane solid. Its constitutive relations may be written as (Soh and Liu, 2005)

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{lij}E_l - h_{lij}H_l \tag{1}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \epsilon_{il}E_l + \beta_{il}H_l \tag{2}$$

$$B_i = h_{ikl}\varepsilon_{kl} + \beta_{il}E_l + \gamma_{il}H_l \tag{3}$$

where σ_{ij}, D_i and B_i are the mechanical stresses, the electric displacements and the magnetic inductions; C_{ijkl}, ϵ_{il} and γ_{il} are the elastic stiffness tensor, the dielectric permitivities and the magnetic permeabilities; e_{lij}, h_{lij} and β_{il} are the piezoelectric, piezomagnetic and magnetoelectric coupling coefficients; and ε_{ij}, E_i and H_i are, respectively, the mechanical strains, the electric field and the magnetic field which are related with the elastic displacements, u_i, the electric potential, ϕ and the magnetic potential, φ, by the following expressions.

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{4}$$

$$E_i = -\phi_{,i} \tag{5}$$

$$H_i = -\varphi_{,i} \tag{6}$$

Eqs. (1)–(3) can be written in a more compact form as

$$\sigma_{iJ} = C_{iJKl}u_{K,l} \tag{7}$$

when the extended notation is considered, in such a way that the displacement vector is extended with the electric potential and the magnetic potential as

$$u_I = \begin{cases} u_i & I = 1,2 \\ \phi & I = 4 \\ \varphi & I = 5 \end{cases} \tag{8}$$

and the stress tensor is extended with the electric displacements and the magnetic inductions as

$$\sigma_{iJ} = \begin{cases} \sigma_{ij} & J = 1,2 \\ D_i & J = 4 \\ B_i & J = 5 \end{cases} \tag{9}$$

whilst the elasticity tensor is extended as

$$C_{iJKl} = \begin{cases} C_{ijkl} & J,K = 1,2 \\ e_{lij} & J = 1,2;\ K = 4 \\ h_{lij} & J = 1,2;\ K = 5 \\ e_{ikl} & J = 4;\ K = 1,2 \\ -\epsilon_{il} & J,K = 4 \\ -\beta_{il} & J = 4;\ K = 5 \\ h_{ikl} & J = 5;\ K = 1,2 \\ -\beta_{il} & J = 5;\ K = 4 \\ -\gamma_{il} & J,K = 5 \end{cases} \tag{10}$$

so that the lowercase (elastic) and uppercase (extended) subscripts adopt values 1, 2 and 1, 2 (mechanical), 4 (electric), 5 (magnetic), respectively.

2.2. Governing equations

The dynamic equilibrium equations for an elastic medium under time-harmonic loading are given by