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Damage analysis and dynamic response of elasto-plastic laminated composite shallow spherical shell under low velocity impact

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ABSTRACT

Based on the elasto-plastic mechanics, the damage analysis and dynamic response of an elasto-plastic laminated composite shallow spherical shell under low velocity impact are carried out in this paper. Firstly, a yielding criterion related to spherical tensor of stress is proposed to model the mixed hardening orthotropic material, and accordingly an incremental elasto-plastic damage constitutive relation for the laminated shallow spherical shell is founded when a strain-based Hashin failure criterion is applied to assess the damage initiation and propagation. Secondly, using the presented constitutive relations and the classical nonlinear shell theory, a series of incremental nonlinear motion equations of orthotropic moderately thick laminated shallow spherical shell are obtained. The questions are solved by using the orthogonal collocation point method, Newmark method and iterative method synthetically. Finally, a modified elasto-plastic contact law is developed to determine the normal contact force and the effect of damage, geometrical parameters, elasto-plastic contact and boundary conditions on the contact force and the dynamic response of the structure under low velocity impact are investigated.

1. Introduction

Fiber-reinforced composite materials are extensively applied to the structures' manufactures in modern industries recently. These composite structures are susceptible to impact which inevitably exists in the transportation and application. So the forecast of the damage and impact response of these structures is of great practical benefit to their design and manufacture.

The impact event is a complex phenomenon which includes the interaction between projectile and structure. Recently, lots of researches on impact have been carried out in experimental and numerical form. When only the elastic contact is in consideration, Yang and Sun (1981) presented the experimental indentation law through static indentation tests on composite laminates; Tam and Sun (1982) developed their own finite program to analyze impact response of composite laminates and performed impact tests by using a pendulum type low velocity impact test system; Sun (1977) applied the modified Hertzian contact law to the dynamic analysis of the composite laminates under impact; Choi and Lim (2004) proposed a linearized contact law in low velocity impact analysis of composite laminates and compared it to the modified Hertzian contact law. When the plastic deformation occurs in contact area as the deflection of the structure under impact increases, the elastic contact law is no longer suitable to model the contact force and a more feasible contact law is needed. So Johnson (1985) applied the Hertzian theory and Von Mises yield criterion to determine the normal force at which the incipient yield occurs in two spheres subjected to a normal load. Chang et al. (1987) proposed the CEB (Chang, Etsion and Bogy) model in the analysis of composite laminates under impact of a sphere. In this model the sphere remains in elastic Hertzian contact until a critical interference is reached, above which volume conservation of the sphere tip is imposed. The contact pressure distribution for the plastic deformed composite laminates structures was assumed to be rectangular and equal to the maximum Hertzian pressure at critical interference. The CEB model suffers from a discontinuity in contact load as well as in the first derivatives of both contact load and contact area at the transition from the elastic to the elasto-plastic regime. Vu-Quoc and Zhang (1999), Vu-Quoc et al. (2001, 2000) made further advance in the theory and numerical analysis, and proposed an accurate elasto-plastic NFD (normal force–displacement) model, which had been experimentally validated in Plantard and Papini (2005). But this model is only suitable to the elasto-perfectly plastic material. So, we attempt in the present paper to establish an elasto-plastic contact law based on the elasto-perfectly plastic NFD (normal force–displacement) model presented in Vu-Quoc et al. (2001). However, only the case of normal impact is investigated in this paper, and the oblique impact, a more general and realistic case, can refer to Vu-Quoc et al. (2004), Zhang

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and Vu-Quoc (2007), in which a tangential elasto-plastic force–displacement model had been developed.

Invisible damages of various kinds are easy to be induced in composite laminates when impacted by projectiles, such as: delamination, matrix crack and fiber breakage. They would heavily reduce the stiffness and life-span of the structures. Experiments show that composite materials are more susceptible to the impact than the metal. So a full comprehension of the damage mechanism for composite laminates under low velocity impact is very important. Collombet et al. (1996) presented some numerical tools to simulate the low velocity impact damage of laminated composite structures. They considered a model of contact-impact based on Lagrange multiplier technique. Matrix cracking is represented by an averaging technique developed on the scale of one finite element. The results showed good agreement between the experimental and numerical damage observations. Choi and Chang (1992) used the dynamic finite element method coupled with failure analysis to predict the threshold of impact damage and initiation of delamination. Hou et al. (2000) predicted the impact damage for laminated composites with implementation of an improved failure criterion, suggesting that delamination was constrained by the through-thickness compression stress; Ganapathy and Rao (1997) predicted the damage in laminated composite plates and in cylindrical/spherical shell panels subjected to low velocity impact. The in-plane damage in the laminates was firstly analyzed by a 2D nonlinear finite element model using laminated composite shell elements with a 48 degree-of-freedom. The in-plane damage was then analyzed by using the Tsai–Wu criterion and maximum stress criteria.

In this paper, an elastic progressive stiffness modification method is established by adopting the strain-based Hashin failure criterion at first, and then an elasto-plastic constitutive relation for orthotropic materials containing damage are built when the yield criterion related to spherical tensor of stress is proposed to describe the mixed hardening of damaged orthotropic materials. Based on the nonlinear classical flat shell theory, the incremental nonlinear motion equations of orthotropic moderately thick shallow spherical shells are obtained. With the previously deduced yield criterion related to spherical tensor of stress, we developed the Vu-Quoc et al. (2000) elasto-perfectly plastic contact model to model the contact force and indentation in the plastic loading phase. Numerical results show the effect of damage, geometrical parameters of the structure and boundary conditions on the contact force and the dynamic response of the structure under low velocity impact.

2. Basic equations

2.1. Incremental nonlinear geometric relations for the laminated composite shallow spherical shell

Consider an axi-symmetrical laminated moderately thick shallow spherical shell with the thickness h, the number of the plies N and base radius a. The shell is impacted by an elastic sphere on the top with a velocity of v_0 (as shown in Fig. 1). Each point in the shells can be denoted with the orthogonal curvilinear coordinates φ, θ, z along the meridional, circumferential and radial/thickness directions, respectively. $z = 0$ denotes the mid-surface and $z = \pm h/2$ denote the inner and outer surfaces of the laminated shallow spherical shell, respectively. The curvature radius of the mid-surface is $R_1 = R_2 = R$, and the Lame coefficients are $A_1 = R, A_2 = R\sin\varphi$. Under the *Timoshenko–Midlin* assumption, the incremental displacement components du, dv, dw of any point in the shell at any time for the axi-symmetrical deformation can be expressed as

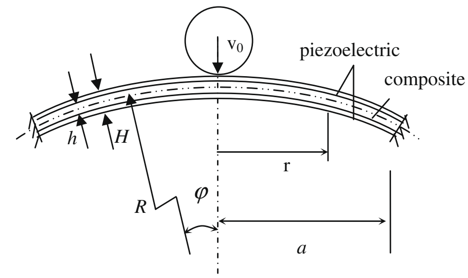

Fig. 1. Geometrical configuration of the shallow spherical shell with fixed boundary.

$$\begin{aligned} du(\varphi,\theta,z,t) &= du^0(\varphi,\theta,t) + zd\psi_1(\varphi,\theta,t) \\ dv(\varphi,\theta,z,t) &= 0 \\ dw(\varphi,\theta,z,t) &= dw^0(\varphi,\theta,t) \end{aligned} \tag{1}$$

where du^0, dw^0 are the incremental displacement components on the mid-surface of the laminated shallow spherical shell; $d\psi_1$ is the incremental independent rotation of the radial section. The nonlinear incremental strain–displacement relations are expressed as follows

$$d\varepsilon_\varphi = d\varepsilon_\varphi^0 + zdk_\varphi^0, \quad d\varepsilon_\theta = d\varepsilon_\theta^0 + zdk_\theta^0, \quad d\varepsilon_{\varphi z} = d\psi_1 + \frac{1}{R}\frac{\partial dw}{\partial \varphi} \tag{2}$$

where $d\varepsilon_\varphi^0, d\varepsilon_\theta^0$ are the incremental strain components on the mid-surface, $dk_\varphi^0, dk_\theta^0$ are the changes of the curvatures on the mid-surface, and

$$\begin{aligned} &d\varepsilon_\varphi^0 = \frac{1}{R}\frac{\partial du^0}{\partial \varphi} - \frac{dw}{R} + \frac{1}{2}(d\omega_2)^2 + \omega_2 \cdot d\omega_2, \\ &d\varepsilon_\theta^0 = \frac{\cot\varphi}{R}du^0 - \frac{dw}{R}, \quad k_\varphi^0 = \frac{1}{R}\frac{\partial d\psi_1}{\partial \varphi}, \quad k_\theta^0 = \frac{\cot\varphi}{R}d\psi_1 \end{aligned} \tag{3}$$

in which ω_2 is rotational component and $\omega_2 = -\frac{1}{2}\left(\frac{1}{R}\frac{\partial w}{\partial \varphi} - \psi_1\right)$, $d\omega_2 = -\frac{1}{2}\left(\frac{1}{R}\frac{\partial dw}{\partial \varphi} - d\psi_1\right)$.

In the analysis of the shallow spherical shell, a new variable r is introduced in the radius of the parallel circle, and then there exist the following relation $r \approx R\varphi, \sin\varphi, \cos\varphi \approx 1$ approximately. Therefore, according to Eq. (3), the incremental strain and rotary angle components can be simply expressed as

$$\begin{aligned} &d\varepsilon_\varphi^0 = \frac{\partial du^0}{\partial r} - \frac{dw}{R} + \frac{1}{2}(d\omega_2)^2 + \omega_2 \cdot d\omega_2, \\ &d\varepsilon_\theta^0 = \frac{du^0}{r} - \frac{dw}{R}, \quad dk_\varphi^0 = \frac{\partial d\psi_1}{\partial r}, \quad dk_\theta^0 = \frac{d\psi_1}{r}, \\ &\omega_2 = -\frac{1}{2}\left(\frac{\partial w}{\partial r} - \psi_1\right), \quad d\omega_2 = -\frac{1}{2}\left(\frac{\partial dw}{\partial r} - d\psi_1\right) \end{aligned} \tag{4}$$

2.2. Incremental elastic damage constitutive relations for orthotropic composite shallow spherical shell

According to Zhang et al. (2006), Chien and Lee (2003), when the orthotropic composite structures are subjected to a low velocity impact, the stiffness coefficients would be reduced when the failure thresholds are reached and would be reduced further as deformation increases. After the stiffness coefficients are firstly degraded at a local point, the stresses may become chaotic while strains are more continuous and would be basis better suited to assess failure. On the basis of the damage model presented by Zhang et al. (2006), and for the axi-symmetrical deformation analysis, three failure symbols, s_1, s_2 and s_3 are defined as follows