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Sliding frictional contact between a rigid punch and a laterally graded elastic medium

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ABSTRACT

Analytical and computational methods are developed for contact mechanics analysis of functionally graded materials (FGMs) that possess elastic gradation in the lateral direction. In the analytical formulation, the problem of a laterally graded half-plane in sliding frictional contact with a rigid punch of an arbitrary profile is considered. The governing partial differential equations and the boundary conditions of the problem are satisfied through the use of Fourier transformation. The problem is then reduced to a singular integral equation of the second kind which is solved numerically by using an expansion–collocation technique. Computational studies of the sliding contact problems of laterally graded materials are conducted by means of the finite element method. In the finite element analyses, the laterally graded half-plane is discretized by quadratic finite elements for which the material parameters are specified at the centroids. Flat and triangular punch profiles are considered in the parametric analyses. The comparisons of the results generated by the analytical technique to those computed by the finite element method demonstrate the high level of accuracy attained by both methods. The presented numerical results illustrate the influences of the lateral nonhomogeneity and the coefficient of friction on the contact stresses.

1. Introduction

Functionally graded materials (FGMs) are multiphase composites that possess smooth spatial variations in the volume fractions of the constituent phases. This broad definition encompasses a wide variety of materials which have gradations in their mechanical, thermal, optical or electrical properties (Khor and Gu, 2000; Park et al., 2000; Cannillo et al., 2007; Yang and Xiang, 2007). In the last decade, the FGM concept has proved to be useful in processing new materials with improved tribological performance. Both experimental and theoretical studies are conducted to examine the feasibility of using FGMs in tribological applications, which generally demand resistance to contact related damage. These studies show that utilization of FGM surfaces in tribological applications leads to improvements in wear resistance (Yue and Li, 2008; Zhang et al., 2008). Furthermore, a controlled gradient in the modulus of elasticity at a surface is shown to eliminate conical cracking that results from Hertzian indentation (Jitcharoen et al., 1998; Pender et al., 2001a,b) and suppress the formation of herringbone cracks under sliding frictional contact (Suresh et al., 1999). The improved tribological characteristics of FGMs paved the way for their potential use in technological applications such as joint prostheses (Mishina et al., 2008) and high performance cutting tools (Nomura et al., 1999).

In the technical literature, various methods, geometries and loading conditions are considered in the contact mechanics analyses of functionally graded materials. Giannakopoulos and Pallot (2000) provided analytical closed form solutions for the two dimensional contact of rigid cylinders on graded elastic substrates. When it is not possible to derive analytical closed form solutions for a contact mechanics problem, generally the problem is reduced to a singular integral equation (SIE) which is solved numerically to compute the required quantities. Dag and Erdogan (2002a) proposed such an SIE based technique to examine the behavior of a surface crack located in an FGM half-plane that is loaded through a frictional flat stamp. Numerical results illustrating the behavior of a surface crack at an FGM surface loaded by a circular stamp are given by the same authors (Dag and Erdogan, 2002b). Guler and Erdogan (2004, 2006, 2007) considered various two dimensional sliding frictional contact problems of FGM coatings by using the singular integral equations. In a series of articles, Ke and Wang developed SIE based solution techniques for two dimensional sliding frictional contact problems of FGMs with arbitrarily varying elastic properties (Ke and Wang, 2006, 2007a) and for partial slip contact problems of FGMs (Ke and Wang, 2007b,c). Yang and Ke

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(2008) studied a two dimensional contact problem for a coating-graded layer–substrate structure loaded by a rigid cylindrical punch. Choi and Paulino (2008) detailed a method based on the singular integral equations which takes into account the effect of frictional heat generation on the contact stress distributions in problems involving FGM coatings and interlayers. Guler (2008) outlined an analytical procedure for the solution of contact problems of thin films and cover plates that are bonded to functionally graded substrates.

In all the studies mentioned in the foregoing paragraph, the material property gradation of the functionally graded medium is assumed to be in the thickness direction and perpendicular to the contact surface. However, in certain technological applications, components that have material property gradations in the lateral direction instead of the thickness direction could also be useful in improving various characteristics of the system behavior. As an example, we consider the bone plates which are used in contact with bones in order to immobilize the fractured segments. It is shown that a bone plate with a lateral gradient in its modulus of elasticity causes less stress-shielding in the bone which facilitates callus formation and improves the bone healing characteristics (Ganesh et al., 2005). The objective in the present study is to develop analytical and computational methods for contact mechanics analysis of functionally graded materials that possess material property gradation in the lateral direction. In the analytical approach, we consider an FGM half-plane in sliding frictional contact with a rigid punch of an arbitrary profile. The shear modulus of the half-plane is assumed to vary exponentially along the lateral direction. The direction of the material property gradation is taken to be perpendicular to the normal of the contact surface. The problem is reduced to a singular integral equation of the second kind which is solved numerically to compute the contact stress distributions. In order to provide more insight into the behavior of the functionally graded materials with lateral gradation, contact mechanics analysis is also conducted by means of the finite element method. This dual approach methodology allows a direct comparison between the analytical and computational results leading to a highly accurate predictive capability. The parametric analyses are performed by considering flat and triangular punch profiles, the former being representative of a complete contact problem and the latter of an incomplete contact problem. The presented results illustrate the influences of the lateral nonhomogeneity and the coefficient of Coulomb friction on the distributions of the contact stresses.

2. Analytical solution

The geometry of the considered contact mechanics problem is depicted in Fig. 1. A functionally graded elastic half-plane is in sliding frictional contact with a rigid punch of an arbitrary profile. The contact area extends from $y = a$ to $y = b$ at the surface $x = 0$. Coulomb's dry friction law is assumed to hold in the contact area hence the tangential force per unit length Q transferred by the contact is taken to be equal to ηP where η is the coefficient of friction and P is the applied normal force per unit length. The shear modulus of the graded medium varies in the lateral direction, i.e. in y-direction. This variation is represented by an exponential function in the following form:

$$\mu(y) = \mu_0 \exp(\gamma y), \tag{1}$$

where μ_0 is the value of the shear modulus at $y = 0$ and γ is a nonhomogeneity constant. The spatial variation of the Poisson's ratio is assumed to be negligible. As a result, the Poisson's ratio ν is considered to be a constant. Under these assumptions, the equations of equilibrium in terms of the displacement components are obtained as follows:

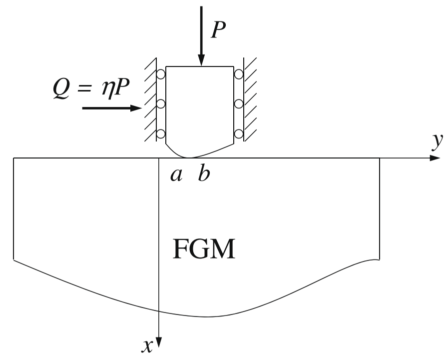

Fig. 1. An FGM half-plane in frictional sliding contact with a rigid punch of an arbitrary profile.

$$(\kappa+1)\frac{\partial^2 u}{\partial x^2} + (\kappa-1)\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 v}{\partial x \partial y} + \gamma(\kappa-1)\frac{\partial u}{\partial y} + \gamma(\kappa-1)\frac{\partial v}{\partial x} = 0, \tag{2}$$

$$(\kappa+1)\frac{\partial^2 v}{\partial y^2} + (\kappa-1)\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \gamma(\kappa+1)\frac{\partial v}{\partial y} + \gamma(3-\kappa)\frac{\partial u}{\partial x} = 0, \tag{3}$$

where u and v are the displacement components in x- and y-directions, respectively, $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for plane stress.

The contact mechanics problem defined above has to be solved by considering the following mixed boundary conditions:

$$\sigma_{xx}(0,y) = \sigma_{xy}(0,y) = 0, \quad -\infty < y < a, \quad b < y < \infty, \tag{4}$$

$$\sigma_{xx}(0,y) = p(y), \quad \sigma_{xy}(0,y) = \eta p(y), \quad a < y < b, \tag{5}$$

$$\frac{\partial u(0,y)}{\partial y} = f(y), \quad a < y < b, \tag{6}$$

$$\int_a^b \sigma_{xx}(0,y)dy = -P, \tag{7}$$

where $p(y)$ is the primary unknown function of the formulation which stands for the contact stress and $f(y)$ is a known function dictated by the profile of the rigid punch. In addition, the solution of the problem must satisfy the regularity conditions at infinity, requiring that all field quantities should be bounded as $(x^2 + y^2) \to \infty$.

The general solutions for the displacement and stress components are obtained by applying Fourier transformation to Eqs. (2) and (3) in y-direction. After applying the Fourier transforms and solving the resulting system of ordinary differential equations, the displacement components are found as

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{j=1}^{2} M_j(\rho)\exp(s_j x + i\rho y)d\rho, \tag{8}$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{j=1}^{2} M_j(\rho)N_j\exp(s_j x + i\rho y)d\rho, \tag{9}$$

where $M_j(\rho)$ $(j = 1,2)$ are unknown functions of ρ, i stands for the imaginary unit $\sqrt{-1}$ and s_j and N_j $(j = 1,2)$ are given as follows:

$$s_1 = -\frac{1}{2}\gamma\sqrt{\frac{3-\kappa}{\kappa+1}} - \frac{1}{2}\sqrt{4\rho^2 + 4i\gamma\rho + \gamma^2\frac{3-\kappa}{\kappa+1}}, \quad \Re(s_1) < 0, \tag{10}$$

$$s_2 = \frac{1}{2}\gamma\sqrt{\frac{3-\kappa}{\kappa+1}} - \frac{1}{2}\sqrt{4\rho^2 + 4i\gamma\rho + \gamma^2\frac{3-\kappa}{\kappa+1}}, \quad \Re(s_2) < 0, \tag{11}$$

$$N_j = -\frac{(\kappa+1)s_j^2 + \gamma(\kappa-1)i\rho - (\kappa-1)\rho^2}{\{2i\rho + \gamma(\kappa-1)\}s_j}, \quad (j = 1,2). \tag{12}$$

The general solutions for the in-plane stress components σ_{xx}, σ_{xy} and σ_{yy} are derived by using Eqs. (8) and (9) and Hooke's Law.