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The thermoelastic Hertzian contact problem

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ABSTRACT

A numerical solution is obtained for the steady-state thermoelastic contact problem in which heat is conducted between two elastic bodies of dissimilar materials at different temperatures with arbitrary quadratic profiles. Thermoelastic deformation causes the initially elliptical contact area to be reduced in size and to become more nearly circular as the temperature difference is increased. There is also a small but identifiable deviation from exact ellipticity at intermediate temperature differences. An approximate analytical solution is obtained, based on approximating the contact area by an ellipse.

1. Introduction

When two conforming bodies are placed in contact, the contact pressure distribution is sensitive to comparatively small changes in surface profile. Thermoelastic deformations, though generally small, can therefore have a significant effect on systems involving contact. For example, Clausing (1966) showed experimentally that the thermal contact resistance between two contacting bodies varied with the transmitted heat flux as a result of thermoelastically driven changes in the extent of the contact area.

If the contacting bodies are small, their surfaces can be approximated by quadratic functions in the vicinity of the contact area and in the absence of thermoelastic deformation, the solution of the elastic contact problem is given by the classical Hertz theory (Johnson, 1985). In particular, the contact area is an ellipse whose ellipticity and orientation are unique functions of the coefficients defining the quadratic surfaces and whose linear dimensions vary with $P^{1/3}$, where P is the contact force.

If the extremities of the two bodies are now raised to different temperatures T_1, T_2, heat will flow through the contact area and the resulting thermoelastic deformation will influence the contact area and the contact pressure distribution. This problem was solved by Barber (1973) for the special case where the bodies are axisymmetric. In this case, the contact area is always circular and its radius a for a given contact force decreases with increasing temperature difference, being given by

$$\hat{a}^3 + \frac{3\Theta\hat{a}^2}{2\pi} = 1, \tag{1}$$

where

$$\hat{a} = \frac{a}{a_H}, \quad \Theta = \frac{(\delta_2 - \delta_1)(T_1 - T_2)KR}{a_H}, \quad \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}, \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, \tag{2}$$

a_H is the radius of the isothermal (Hertzian) elastic contact area for the same contact force P, R_1, R_2 are the radii of the two contacting bodies and the distortivity δ is defined as

$$\delta = \frac{\alpha(1+\nu)}{K}, \tag{3}$$

where α, ν, K are the coefficient of thermal expansion, Poisson's ratio and thermal conductivity. This solution is strictly only applicable when the heat flows into the body with the higher distortivity and hence the dimensionless parameter $\Theta > 0$, since for the opposite direction of heat flow, a small annulus of imperfect thermal contact is developed at the edge of the contact area (Barber, 1978; Kulchytsky-Zhyhailo et al., 2001).

In this paper, we use a numerical method to determine the effect of thermoelastic deformation for the more general Hertzian case where the bodies have general quadratic shapes and the isothermal contact area is elliptical. We shall show that the contact area becomes smaller and also more nearly circular as the temperature difference is increased. We shall also develop an approximate analytical solution to the problem, using an approach proposed by Yevtushenko and Kulchytsky-Zhyhailo (1996).

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2. Statement of the problem

We consider the problem in which two thermally conducting elastic bodies are pressed together by a force P, whilst their extremities are maintained at temperatures T_1 and T_2, respectively. Frictionless contact conditions are assumed and heat flow between the elastic bodies is only permitted to take place by conduction through the contact area A. As in the axisymmetric case, we restrict attention to the case where the heat flows into the more distortive material and hence $(T_1 - T_2)(\delta_1 - \delta_2) < 0$.

2.1. The heat conduction problem

The temperature at the point defined by coordinates (x, y) on the surface of body i can be written as

$$T_i(x,y) = \frac{1}{2\pi K_i} \int\int \frac{q(\xi,\eta)d\xi d\eta}{r} + T_i \tag{4}$$

where q is the heat flux directed into the body and

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2}. \tag{5}$$

In the absence of surface tractions, this heat flux would also cause thermoelastic displacement w_i in the inward normal direction given by (Barber, 1971)

$$w_i(x,y) = \frac{\delta_i}{2\pi} \int\int q(\xi,\eta)\ln(r)d\xi d\eta, \tag{6}$$

where we have omitted a rigid-body displacement.

Continuity of heat flux and temperature at the contact area then leads to the integral equation

$$\Delta T \equiv T_1 - T_2 = \frac{1}{2\pi K} \int\int_A \frac{q(\xi,\eta)d\xi d\eta}{r}, \tag{7}$$

where K is defined in Eq. (2). This equation serves to determine the heat flux q, which is here taken as positive in the direction from body 1 to body 2. Once q is determined, the differential thermoelastic expansion can then be determined from Eq. (6) as

$$w_1(x,y) + w_2(x,y) = \frac{(\delta_2 - \delta_1)}{2\pi} \int\int q(\xi,\eta)\ln(r)d\xi d\eta. \tag{8}$$

2.2. The contact problem

We suppose that the two contacting bodies have profiles defined by the functions $g_1(x,y), g_2(x,y)$, as shown in Fig. 1, so that the initial gap between the undeformed bodies is

$$g_0(x,y) = g_1(x,y) + g_2(x,y). \tag{9}$$

As in the Hertzian theory, we assume that the contact area is sufficiently small to permit this expression to be represented by the quadratic function

$$g_0(x,y) = \frac{x^2}{2R_I} + \frac{y^2}{2R_{II}}, \tag{10}$$

where R_I, R_{II} are the principal radii of curvature of the combined profile, as defined by Eq. (4.3) of Johnson (1985).

If the bodies are now pressed together, a contact pressure $p(x,y)$ will be developed in the contact area, which will generate elastic normal surface displacements

$$u_i(x,y,0) = \frac{1-\nu_i^2}{\pi E_i} \int\int \frac{p(\xi,\eta)d\xi d\eta}{r} \tag{11}$$

directed into the respective bodies $i = 1, 2$, where E_i is Young's modulus of the contacting body i.

The final gap between the bodies is given by

$$g(x,y) = g_0(x,y) + u_1(x,y) + u_2(x,y) + w_1(x,y) + w_2(x,y) + d, \tag{12}$$

where d is an unknown rigid body displacement. The contact problem can then be stated by noting that the gap is zero by definition in the contact area and positive outside, leading to the unilateral contact problem

$$g(x,y) = 0, \quad p(x,y) > 0, \quad (x,y) \in A, \tag{13}$$
$$p(x,y) = 0, \quad g(x,y) > 0, \quad (x,y) \notin A. \tag{14}$$

Combining Eqs. (8), (10)–(14), we then have

$$\begin{aligned} &\frac{1}{\pi E^*} \int\int_A \frac{p(\xi,\eta)d\xi d\eta}{r} + \frac{(\delta_2 - \delta_1)}{2\pi} \int\int_A q(\xi,\eta)\ln(r)d\xi d\eta \\ &\quad = d - g_0(x,y), \quad (x,y) \in A \\ &\quad > d - g_0(x,y), \quad (x,y) \notin A, \end{aligned} \tag{15}$$

where

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}. \tag{16}$$

The corresponding total force is then given by

$$P = \int\int_A p(\xi,\eta)d\xi d\eta. \tag{17}$$

The integral equations (7) and (15) serve to determine the heat flux q and the contact pressure p, whilst Eq. (17) and the inequality in Eq. (15) determine the extent of the contact area A.

2.3. Dimensionless formulation

The number of independent parameters can be reduced by using an appropriate dimensionless representation. We first define two length scales R, a_H through the relations

$$\frac{2}{R} = \frac{1}{R_I} + \frac{1}{R_{II}}, \quad a_H = \sqrt[3]{\frac{3PR}{4E^*}} \tag{18}$$

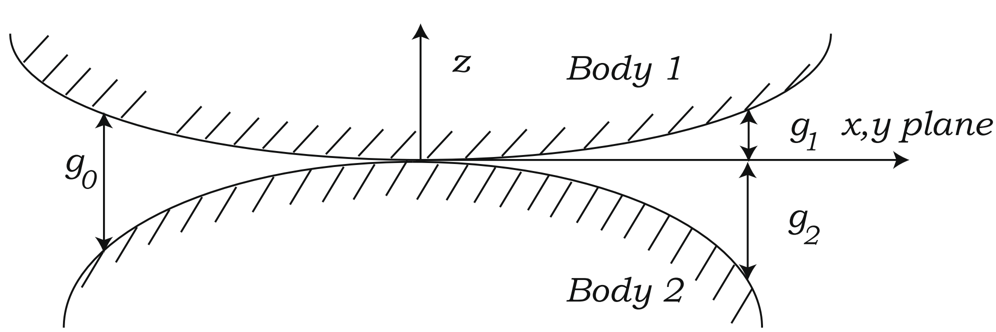

Fig. 1. Initial gap between two bodies.