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Periodic steady state response of large scale mechanical models with local nonlinearities

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ABSTRACT

Long term dynamics of a class of mechanical systems is investigated in a computationally efficient way. Due to geometric complexity, each structural component is first discretized by applying the finite element method. Frequently, this leads to models with a quite large number of degrees of freedom. In addition, the composite system may also possess nonlinear properties. The method applied overcomes these difficulties by imposing a multi-level substructuring procedure, based on the sparsity pattern of the stiffness matrix. This is necessary, since the number of the resulting equations of motion can be so high that the classical coordinate reduction methods become inefficient to apply. As a result, the original dimension of the complete system is substantially reduced. Subsequently, this allows the application of numerical methods which are efficient for predicting response of small scale systems. In particular, a systematic method is applied next, leading to direct determination of periodic steady state response of nonlinear models subjected to periodic excitation. An appropriate continuation scheme is also applied, leading to evaluation of complete branches of periodic solutions. In addition, the stability properties of the located motions are also determined. Finally, representative sets of numerical results are presented for an internal combustion car engine and a complete city bus model. Where possible, the accuracy and validity of the applied methodology is verified by comparison with results obtained for the original models. Moreover, emphasis is placed in comparing results obtained by employing the nonlinear or the corresponding linearized models.

1. Introduction

There are many occasions in engineering practice, where there is a strong interest in determining and studying the long term behavior of a structural or mechanical system, which is subjected to periodic excitation. In such cases, the main interest lies mostly in locating and investigating periodic steady state response. A typical example is the production of response spectra, which are used in order to detect structural or acoustical resonances (Craig, 1981; Kropp and Heiserer, 2003) or to predict fatigue failure of critical structural parts (Lutes and Sarkani, 1997; Benasciutti and Tavo, 2005). When the dynamical system resulting after the modeling possesses linear characteristics, there are standard methods that can lead to this information (Rao, 1990). In cases where the dimension of the models considered becomes excessive, due to the many geometrical details that need to be included in order to cover critical design needs, the volume of the calculations can become prohibitive quite frequently. In such cases, application of appropriate dimension reduction methods in either the time or the frequency domain becomes necessary (Craig, 1981; Cuppens et al., 2000). However, more difficulties arise when nonlinearities are present. In particular, for small scale nonlinear systems subjected to periodic external excitation there is a lot of analytical and numerical work referring to their long term response (Doedel, 1986; Nayfeh and Balachandran, 1995). Among other things, it is well known by now that these systems can exhibit many types of periodic motion as well as more complex behavior, including quasi-periodic and chaotic response. On the other hand, little is still available on capturing long term dynamics for large scale nonlinear systems (Fey et al., 1996; Verros and Natsiavas, 2002).

The main objective of the present work is to develop and apply a systematic methodology leading to a direct determination of periodic steady state response of periodically excited complex mechanical systems. Here, the term complex refers to two characteristic properties of the class of systems examined. The first level of complexity is related to the large number of the corresponding equations of motion. In fact, the detailed geometrical discretization of some of the structural substructures, based on the application of the finite element method (Zienkiewicz, 1986), leads to a large number of equations of motion. As a consequence, in many cases it may not be feasible even to carry over the evaluation of the dynamic quantities needed for the transformations leading to the classical dimension reduction methods (Fey et al., 1996; Chen

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et al., 1998). The second level of complexity is related to the nonlinearities associated with the system response. This poses severe restrictions in the applicability of most of the available and commonly employed methods. On the positive side, a good feature of the class of systems examined is that the nonlinear characteristics are associated with a relatively small number of degrees of freedom of the class of dynamical systems examined.

Based on the localized nature of the nonlinear action, the basic idea of this work is to first reduce the dimension of all the linear components of the structural system examined by applying an appropriate coordinate transformation. In particular, the reduction method applied is based on an automatic multi-level substructuring (Bennighof et al., 2000; Bennighof and Lehoucq, 2004). Apart from increasing the computational efficiency and speed, the reduction of the system dimensions makes amenable the subsequent application of numerical techniques for determining the dynamic response of complex systems, which are applicable and efficient for low order systems. For instance, this method has already been applied successfully to the solution of the real eigenproblem and the prediction of periodic response of large scale linear models with nonclassical damping (Kim and Bennighof, 2006; Papalukopoulos and Natsiavas, 2007), large order gyroscopic systems (Elssel and Voss, 2006) and broadband vibro-acoustic simulations of vehicle models (Kropp and Heiserer, 2003). In addition, the same method has also facilitated determination of the transient response of large scale nonlinear models (Papalukopoulos and Natsiavas, 2007; Theodosiou and Natsiavas, 2009).

In the present work, the same multi-level substructuring method is coupled with an appropriate numerical procedure in order to determine periodic steady state motions of the models examined, resulting in response to periodic excitation in a direct manner (Doedel, 1986). This coupling takes into account and exploits the characteristics of the general class of mechanical systems considered. Moreover, a suitable method is also applied, based on classical Floquet theory and leading to determination of the stability properties of the located periodic motions (Nayfeh and Mook, 1979). Finally, the methodology developed is complemented by a continuation method, yielding complete branches of periodic motions over a specified frequency interval (Ragon et al., 2002). The final outcome is expected to provide valuable information and insight to engineers dealing with the analysis and design of complex mechanical systems.

The validity and effectiveness of the methodology developed is illustrated and verified by presenting a selected set of numerical results. More specifically, some typical results are presented first for a detailed finite element model of a crankshaft, belonging to an internal combustion engine of a commercial car. Then, response spectra for a quite involved city bus model are also determined and presented. The results obtained are useful in assessing the dynamic response of the mechanical systems examined. In both cases, particular emphasis is placed on identifying and evaluating effects caused by the nonlinear action in the dynamics, in comparison with similar predictions of the linear theory.

The organisation of this paper is as follows. First, the characteristics of the class of mechanical models examined are presented in the following section. Then, the basic steps of the methodology developed, including both the application of the coordinate reduction and the direct determination of the periodic steady state response, are summarised in the third and fourth section, respectively. Next, the dynamic response of two example models subjected to periodic external excitation is investigated. Where possible, the accuracy and effectiveness of the present methodology is established by comparison of results obtained for the reduced and the corresponding complete dynamical models. The work is completed by summarizing the highlights in the last section.

2. Class of mechanical models examined

Accurate prediction of the dynamic response of mechanical systems requires frequently the development and examination of geometrically detailed dynamical models. On the one hand, this leads to a quite large set of equations of motion. The situation becomes more complicated when the systems are forced to operate in conditions involving activation of nonlinear characteristics. On the other hand, the information extracted from the systematic prediction of the dynamic response is essential and valuable in performing efficiently other useful studies, as well, related to fatigue, acoustics, identification, optimization and control of a system.

Typically, a complex mechanical system is composed of several structural components. Based on strict design requirements on accuracy, these components are usually discretized geometrically by a relatively large number of finite elements (Bathe, 1982; Hughes, 1987). In many cases, this gives rise to a dynamical system with an excessive number of degrees of freedom. For all practical purposes, the model of each mechanical substructure can be assumed to possess linear properties. However, the elements connecting the system substructures are typically characterized by strong nonlinear action. Taking all the above into account, the equations of motion of the general class of dynamical systems considered in this work can be put in the compact matrix form

$$\widehat{M}\,\underline{\ddot{x}} + \widehat{C}\,\underline{\dot{x}} + \widehat{K}\,\underline{x} + \widehat{\underline{p}}(\underline{x},\underline{\dot{x}}) = \widehat{\underline{f}}(t), \tag{1}$$

where all the unknown coordinates are included in the vector

$$\underline{x}(t) = (\,x_1 \quad x_2 \quad \ldots \quad x_n\,)^T,$$

while $\widehat{M}$, $\widehat{C}$ and $\widehat{K}$ are the mass, damping and stiffness matrix of the system, respectively. These quantities include contribution from all the structural components of the system. Moreover, the elements of vector $\widehat{\underline{p}}(\underline{x},\underline{\dot{x}})$ include the contribution of the nonlinear terms arising from the action of the coupled dynamical system, while vector $\widehat{\underline{f}}(t)$ represents the action arising from the external forcing.

Besides the large number of degrees of freedom, the level of difficulty in determining the dynamic response of the class of systems examined increases considerably when nonlinearity effects become important. However, an attractive feature of these systems is that their nonlinearities usually appear mainly at a relatively small number of places, involving a small portion of the degrees of freedom. This makes possible the application of special techniques, which are appropriate for systems with local nonlinearities. Namely, for such systems it is possible to reduce significantly the number of the original degrees of freedom by applying suitable coordinate reduction methods (Craig, 1981; Fey et al., 1996; Verros and Natsiavas, 2002). Apart from increasing the computational efficiency and speed, the reduction of the system dimensions makes amenable the application of several numerical techniques, which are applicable and efficient for low order dynamical systems.

The dimension of the class of systems examined in the present work can be so high, that ordinary coordinate reduction methods may not be numerically efficient to apply. For this reason, a special coordinate reduction method is applied instead (Bennighof et al., 2000; Bennighof and Lehoucq, 2004), whose basic steps are presented in the following section. This reduction leads to a substantial acceleration of the subsequent calculations. More specifically, the emphasis of the present study is placed on locating periodic steady state motions, when the mechanical models examined are subjected to periodic external excitation.

In general, the long term response of a nonlinear dynamical system to periodic excitation can be either regular (periodic or quasi-periodic) or irregular (chaotic) (Nayfeh and Balachandran, 1995). Typically, such motions are determined by a direct integration of the equations of motion, starting from some selected set of initial