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On non-physical response in models for fiber-reinforced hyperelastic materials

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ABSTRACT

Soft biological tissues are sometimes composed of thin and stiff collagen fibers in a soft matrix leading to a strong anisotropy. Commonly, constitutive models for quasi-incompressible materials, as for soft biological tissues, make use of an additive split of the Helmholz free-energy into a volumetric and a deviatoric part that is applied to the matrix and fiber contribution. This split offers conceptual and numerical advantages. The purpose of this paper is to investigate a non-physical effect that arises thereof. In fact, simulations involving uniaxial stress configurations reveal volume growth at rather small stretches. Numerical methods such as the Augmented Lagrangian method might be used to suppress this behavior. An alternative approach, proposed here, solves this problem on the constitutive level.

1. Introduction

Soft matter often exhibits complex mechanical behavior related to inhomogeneity, anisotropy and (near) incompressibility. This is the case for soft biological tissues that are sometimes characterized by strong anisotropy due to the presence of thin and stiff collagen fibers in a soft extracellular matrix. Examples for this are ligaments and tendons, arterial walls or the annulus fibrosus. Under physiological conditions these tissues might undergo large deformations, necessitating a mechanical description in the framework of nonlinear continuum mechanics. Numerical methods are often used to simulate and understand their mechanical behavior. This has motivated a number of recent studies on constitutive modeling approaches for fiber-reinforced hyperelastic materials.

Classical hyperelastic models, such as the polynomial forms (Rivlin and Saunders, 1951) or the Ogden model (Ogden, 1972) were designed for the modeling of isotropic materials. Later, these models were modified to allow for an additive split of the Helmholz free-energy into a volumetric and a deviatoric part, to some extent motivated by the multiplicative decomposition of the deformation gradient introduced by Flory (1961). It was shown by Ehlers and Eipper (1998) that, if this split is applied to materials not restricted to (nearly) incompressible behavior, it might lead to unphysical responses in uniaxial tension.

Anisotropy itself can be modeled by explicitly including a fiber contribution in the strain energy formulation. Thereby, the idea of the additive split might also be applied to the fiber contribution. To our best knowledge, the first fiber-reinforced model using this additive split assumption for (nearly) incompressible materials was proposed by Weiss et al. (1996). In 2000, Holzapfel et al. published a model that has since then become very popular in simulations involving anisotropic biological tissues.

The purpose of this paper is to investigate a non-physical effect in the application of the additive split to fiber-reinforced (nearly) incompressible materials. In fact, numerical simulations involving uniaxial stress configurations lead to volume growth when using the models by Weiss et al. (1996) or Holzapfel et al. (2000). To prevent this behavior, numerical costly methods such as the Augmented Lagrangian method (Simo and Taylor, 1991) might be used.

An alternative approach, as proposed here, is to avoid the multiplicative decomposition of the deformation gradient in the fiber contribution. Although several authors (Holzapfel et al., 2004; Schröder et al., 2005) have already proposed model formulations without this decomposition, no statements were found in the papers that this was motivated by the problems reported in the present work. In general, no criterion is proposed for or against this decomposition being applied to the fiber contributions to the free-energy. Rather it seems the invariants and their modified equivalents are considered “to be equivalent” in case of incompressibility. Though it will be shown in this paper that the choice of either one has an influence on the stress and stiffness terms of the fiber contribution.

Our results are also compared with the predictions from the model proposed by Rubin and Bodner (2002), which avoids the

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volumetric split for the fiber contribution and uses an additive split in the log-energies.

2. Methods

2.1. Continuum mechanics background

For Green elastic materials the Cauchy (true) stress $\boldsymbol{\sigma}$ and the spatial stiffness $\mathbb{c}$ can be obtained from a strain energy density function Ψ (the Helmholz free-energy per unit reference volume) by

$$\boldsymbol{\sigma} = \frac{2}{J}\boldsymbol{\chi}_*\left[\frac{\partial\Psi[\mathbf{C}]}{\partial\mathbf{C}}\right], \quad \mathbb{c} = \frac{4}{J}\boldsymbol{\chi}_*\left[\frac{\partial^2\Psi[\mathbf{C}]}{\partial\mathbf{C}\partial\mathbf{C}}\right], \tag{1}$$

respectively.[1] Here, $\boldsymbol{\chi}^*[]$ is the push-forward operator to bring the material derivatives to the spatial configuration. Ψ depends solely on the right Cauchy–Green deformation tensor $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$, where $\mathbf{F}$ is the deformation gradient and $()^{\mathrm{T}}$ depicts the usual transpose of a second order tensor.

Anisotropy due to fibers can be included by use of structural tensors

$$\mathbf{A}^{(i)} = \zeta^{(i)} \otimes \zeta^{(i)}, \tag{2}$$

where $\zeta^{(i)}$ is a normalized vector describing the orientation of the ith fiber family in the reference configuration. The principles of material frame indifference and invariance under transformations that respect the material symmetries lead to a representation of Ψ that depends only on the so called mixed invariants of $\mathbf{C}$ and $\mathbf{A}^{(i)}$:

$$\Psi = \Psi[I_1,\ I_2,\ I_3,\ I_4^{(i)}] \tag{3}$$

with

$$I_1 = \mathrm{tr}[\mathbf{C}], \tag{4a}$$

$$I_2 = \frac{1}{2}\left((\mathrm{tr}[\mathbf{C}])^2 - \mathrm{tr}[\mathbf{C}^2]\right), \tag{4b}$$

$$I_3 = \det[\mathbf{C}] = J^2, \tag{4c}$$

$$I_4^{(i)} = \zeta^{(i)} \cdot \mathbf{C}\zeta^{(i)} = \mathbf{C} : \mathbf{A}^{(i)} = (\zeta^{(i)})^2. \tag{4d}$$

$J = \det[\mathbf{F}]$ denotes the volume ratio and $I_4^{(i)}$ represents the square of the stretch $\zeta^{(i)}$ of fiber family i. Note that the set of invariants I_γ, $\gamma = 1,\ldots,4$ is not the full set of invariants of the tensors $\mathbf{C}$ and $\mathbf{A}^{(i)}$ (Antman, 2005) but includes the commonly used ones.

The right Cauchy–Green tensor $\mathbf{C}$ can be split multiplicatively into a deviatoric part $\overline{\mathbf{C}}$ and a spherical part $\mathbf{C}_{\mathrm{vol}}$ (Flory, 1961):

$$\mathbf{C} = \mathbf{C}_{\mathrm{vol}}\overline{\mathbf{C}} = J^{2/3}\mathbf{I}\overline{\mathbf{C}}. \tag{5}$$

Using this split it is common to write the strain energy Ψ in an uncoupled form

$$\Psi = U[J] + \overline{\Psi}\left[\bar{I}_1,\ \bar{I}_2,\ \bar{I}_4^{(i)}\right] \tag{6}$$

in which U is the response of the material to volume changes and $\overline{\Psi}$ depends only on the distortional part of the deformation. $\bar{I}_1$, $\bar{I}_2$ and $\bar{I}_4^{(i)}$ are the invariants of the deviatoric part $\overline{\mathbf{C}}$ of the right Cauchy–Green deformation tensor $\mathbf{C}$. It should be noted that, in general, the invariant $\bar{I}_4^{(i)}$ no longer represents the square-stretch of fiber family i; rather,

$$\bar{I}_4^{(i)} = J^{-2/3}(\zeta^{(i)})^2. \tag{7}$$

The split (6) enables distinct modeling of high resistance to volumetric deformation and low resistance to distortional deformation (as in the case of nearly incompressible materials). Sansour (2008) shows that the additive split is a consequence of the assumption that the pressure is solely a function of J.

In case of fiber-reinforced materials $\overline{\Psi}$ can be divided into $\overline{\Psi}_{gs}$ and $\overline{\Psi}_f$ for the energy contributions of the matrix ("ground substance") and the fibers, respectively:

$$\Psi[\mathbf{C}, \mathbf{A}_i] = U[J] + \overline{\Psi}_{gs}[\bar{I}_1, \bar{I}_2] + \overline{\Psi}_f[\bar{I}_4^{(i)}]. \tag{8}$$

In the following, three structural, anisotropic constitutive models are presented. For ease of reading their notation has been unified from their original presentations. The closed forms of the stress and material stiffness tensors are given in the original papers.

2.1.1. Model proposed by Weiss et al. (1996)

Weiss et al. (1996) suggested a constitutive model that uses a strain energy function of the form (8). The individual contributions are given by

$$U[J] = \frac{\kappa}{2}(\ln[J])^2, \tag{9a}$$

$$\overline{\Psi}_{gs}[\bar{I}_1, \bar{I}_2] = w_1(\bar{I}_1 - 3) + w_2(\bar{I}_2 - 3), \tag{9b}$$

$$\overline{\Psi}_f[\bar{I}_4^{(i)}] = \frac{w_3}{w_4}\sum_{i=1}^{n}\left(\mathrm{e}^{w_4(\bar{I}_4^{(i)}-1)} - \bar{I}_4^{(i)\,w_4}\right) \tag{9c}$$

where κ is the small strain bulk modulus and the w_γ, $\gamma = 1,\ldots,4$, are material parameters. w_4 is introduced in the present work as an additional parameter that allows one to shift the onset of the fiber response and thus to fit the model to the selected uniaxial response ($w_4 = 1$ [−] retrieves the original model). Note that the contribution of the ground substance is described by a Mooney–Rivlin model (a special case of the polynomial materials Rivlin and Saunders, 1951) with the small strain shear modulus $\mu = 2(w_1 + w_2)$.

2.1.2. Model proposed by Holzapfel et al. (2000)

Holzapfel et al. (2000) proposed a model according to Eq. (8) for modeling the arterial wall. The explicit forms of the energy contributions are given by

$$U[J] = \frac{\kappa}{2}(J-1)^2, \tag{10a}$$

$$\overline{\Psi}_{gs}[\bar{I}_1] = \frac{\mu}{2}(\bar{I}_1 - 3), \tag{10b}$$

$$\overline{\Psi}_f[\bar{I}_4^{(i)}] = \frac{h_1}{2h_2}\sum_{i=1}^{n}\left(\mathrm{e}^{h_2(\bar{I}_4^{(i)}-1)^2} - 1\right), \tag{10c}$$

where κ, μ, h_1 and h_2 are the small strain bulk and shear moduli and two material parameters, respectively. The ground substance is modeled as a Neo-Hookean material.

In both these models the fibers are thought to be active only in tension and therefore all contributions (energy, stress and stiffness) of fiber family i are set to zero if $\bar{I}_4^{(i)} < 1$.

2.1.3. Model proposed by Rubin and Bodner (2002)

The model presented by Rubin and Bodner (2002) uses a different strain energy function:

$$\Psi = \frac{p}{2}\left(\exp^{p^{-1}\widehat{\Psi}} - 1\right), \tag{11}$$

where p is a material constant. $\widehat{\Psi}$ is the sum of four contributions

$$\widehat{\Psi} = \widehat{\Psi}_1[J] + \widehat{\Psi}_2[\bar{I}_1] + \widehat{\Psi}_3[\zeta_i] + \widehat{\Psi}_4[\alpha_1]. \tag{12}$$

The functions $\widehat{\Psi}_\gamma, \gamma = 1, \ldots, 4$, characterize the response to total dilatation, total distortion, fiber stretching and distortional deformation of a dissipative component, respectively. $\widehat{\Psi}_4$ will be neglected here, reducing the model to pure elastic response:

[1] Our notation of tensor operators follows Holzapfel (2000).