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Discrete material optimization of vibrating laminated composite plates for minimum sound radiation

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ABSTRACT

This paper deals with vibro-acoustic optimization of laminated composite plates. The vibration of the laminated plate is excited by time-harmonic external mechanical loading with prescribed frequency and amplitude, and the design objective is to minimize the total sound power radiated from the surface of the laminated plate to the surrounding acoustic medium. Instead of solving the Helmholtz equation for evaluation of the sound power, advantage is taken of the fact that the surface of the laminated plate is flat, which implies that Rayleigh's integral approximation can be used to evaluate the sound power radiated from the surface of the plate. The novel Discrete Material Optimization (DMO) formulation has been applied to achieve the design optimization of fiber angles, stacking sequence and selection of material for laminated composite plates. Several numerical examples are presented in order to illustrate this approach.

1. Introduction

Composite materials like fiber reinforced polymers (FRPs) are being used increasingly in aerospace vehicles, maritime carriers, wind turbine blades, and various mechanical equipment where high strength, high stiffness and low weight are important properties. In such applications, the FRPs are usually stacked in a number of layers, each consisting of strong fibers bonded together by a resin, to form a laminate. In addition, laminated sandwich structures may also consist of layers made of foam material. When these composite structures are used in dynamic environments, vibration control and noise reduction become of great technical significance. In the present paper, this design objective will be considered in the form of minimizing the sound radiation from a laminated composite plate. This is accomplished by optimizing simultaneously the laminates in terms of proper choice of material, stacking sequence and fiber orientation. Such advanced optimization of laminates has only recently become possible via development of optimization approaches such as the method of Discrete Material Optimization (DMO) in the papers (Stegmann and Lund, 2005; Lund and Stegmann, 2005), and practically feasible via new developments in the manufacture of composite materials and structures.

Excellent textbooks on optimization for passive design of structural–acoustic systems against vibration and noise (Koopmann and Fahnline, 1997; Kollmann, 2000), and the proceedings of an IUTAM Symposium on design for quietness (Munjal, 2002), were already published about 10 years ago. These publications as well as papers like (Pedersen, 1982; Olhoff and Parbery, 1984; Bendsøe and Olhoff, 1985; Christensen et al., 1998a,b; Sorokin et al., 2006; Bös, 2006) did not specially address composite structure applications, but provide an overview and contain references to the area of design optimization with respect to acoustic criteria in general.

However, as mentioned above, structural–acoustic optimization of composite structures has received increasing attention with the extensive application of composite materials in recent years, and in terms of optimization of composite structures with respect to acoustic criteria, we may refer the reader to the review article (Denli and Sun, 2007) and the bibliography (Mackerle, 2003), and a large number of papers cited therein. As examples of various types of problems of optimum structural–acoustic design with composite materials, we may refer to (Hufenbach et al., 2001; Thamburaj and Sun, 2002; Chen et al., 2005; Yamamoto et al., 2008; Jensen, 2009).

Relative to sizing or shape optimization, structural topology optimization generally yields a more efficient conceptional/configurational design at the stage of initial design. In 1988, Bendsøe and Kikuchi (1988) implemented the topology optimization of continuum structures via a homogenization method and laid the foundation of modern structural topology optimization, and this type of optimization has been an extremely active area of research since then. For the state-of-the-art in topology optimization the reader is referred to the review article (Eschenauer and Olhoff, 2001), the exhaustive textbook (Bendsøe and Sigmund, 2003), the paper

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(Bendsøe et al., 2005), and the proceedings from an IUTAM Symposium (Bendsøe et al., 2006).

Up to now, the method of topology optimization has been mainly applied to three classes of problems within the area of structural–acoustic design, however with no particular focus on composite structure applications. The first class of problems encompasses maximization of intrinsic properties of the structures like fundamental and higher order eigenfrequencies, gaps between two consecutive eigenfrequencies, and phononic band gaps, see, e.g. (Dias and Kikuchi, 1992; Bendsøe and Diaz, 1994; Ma et al., 1995; Kosaka and Swan, 1999; Krog and Olhoff, 1999; Pedersen, 2000; Sigmund, 2001; Tcherniak, 2002; Jensen, 2003; Sigmund and Jensen, 2003; Olhoff and Du, 2005; Diaz et al., 2005; Jensen and Pedersen, 2006; Halkjær et al., 2006; Du and Olhoff, 2007b; Mendonca et al., 2009; Akl et al., 2009). The second class of problems comprises minimization of the dynamic compliance, i.e., maximization of the dynamic stiffness, of structures subjected to forced vibration by time-harmonic external mechanical loading of given frequency (or frequency range), see, e.g. (Min et al., 1999; Jog, 2002a,b; Calvel and Mongeau, 2005; Jensen, 2007; Olhoff and Du, 2010). Finally, the third class of structural–acoustic topology design problems includes minimization of the acoustic power radiated from the structural surface(s) into a surrounding or interior acoustic medium like air, when the structure is subjected to forced vibration by time-harmonic external mechanical loading of given frequency or frequency range, see, e.g., the papers (Luo and Gea, 2003; Wadbro and Berggren, 2006; Yoon et al., 2007; Du and Olhoff, 2007a, 2010; Dühring et al., 2008; Olhoff and Du, 2008).

It is worth noting (Olhoff and Du, 2008) that the design objectives of the above-mentioned three classes of structural–acoustic topological design optimization problems generally have the same mechanical effect. This effect consists in driving the nearest resonance frequencies of the structure as far away as possible from a given external excitation frequency, or a band of excitation frequencies; this way structural resonance phenomena with high vibration and noise emission levels are avoided by the optimization.

Through structural topology optimization, composite structures can be tailored to optimally achieve certain objectives. In the present paper, the objective is to minimize the acoustic power radiated from vibrating composite structures. An efficient and reliable numerical design tool is very important in order to realize the topology optimization of vibrating laminated composite plates, and the novel Discrete Material Optimization (DMO) formulation proposed in Stegmann and Lund (2005) and Lund and Stegmann (2005) has been applied to achieve this goal.

The optimization of fiber orientation in composite laminates was first considered by Rasmussen (1979), see Niordson and Olhoff (1979). Several other methods have already been proposed to realize optimum fiber orientation problems for orthotropic materials: optimality criteria for orientational design (Pedersen, 1989, 1991; Cheng and Pedersen, 1997), parameterization based on lamination parameters (Miki and Sugiyama, 1993), a general approach of forcing convexity of ply angle optimization based on lamination parameters (Foldager et al., 1998), and an energy based method (Luo and Gea, 1998) for the optimum orientation problem. The optimization of fiber orientation and concentration of one or two fiber fields in composite laminates is dealt with in Thomsen and Olhoff (1990), Thomsen (1991), where cross-ply fiber reinforcement is allowed for. As opposed to these methods of optimization of fiber orientation, DMO is based on ideas from multiphase topology optimization (Sigmund and Torquato, 1997) to achieve a parameterization. Thus, DMO lends itself to the methodology of topology optimization, and the method has proven to reduce the risk of obtaining local optimum solutions for fiber orientations, and to be able to solve simultaneously the optimization of materials, stacking sequence and fiber orientations, using a predefined set of discrete candidate materials and fiber angles as design variables at element level.

It is assumed in this paper that air is the acoustic medium and that a feedback coupling between the acoustic medium and the structure can be neglected. Rayleigh's integral is used for computation of the sound power radiated from the structural surface instead of solving the Helmholtz integral equation. This implies that the computational cost of the structural–acoustic analysis can be considerably reduced, which is very efficient for the design optimization. It has been proved that Rayleigh's integral provides a very good approximation of the sound pressure distribution along a relatively flat structural surface, see Lax and Feshbach (1947), Herrin et al. (2003), and Du and Olhoff (2007a).

The organization of the remainder of this paper is as follows. In Section 2, the problem of structural topology optimization of laminated composite plates subject to given amounts of the constituents is formulated for the objective of minimizing the sound power radiated from the vibrating structural surface into the acoustic medium. Then Rayleigh's integral is introduced to calculate the sound power flow from the structural surface, which leads to a simplified optimization formulation of the current problem. Section 3 introduces the parameterization for Discrete Material Optimization (DMO) and discusses penalty functions for DMO. Section 4 presents a DMO convergence measure. Section 5 deals with the sensitivity analysis required for the numerical optimization algorithm. Section 6 outlines several numerical examples with different excitation frequencies in order to validate the proposed method, including single-layer, multi-layer laminated composite plates, and laminated sandwich plates consisting of layers made of FRPs and foam material. In Section 7, two isotropic materials are introduced together with unidirectional fiber material as candidate materials in single-layer plate design. Finally, a section with conclusions closes the paper.

2. Minimization of sound power radiation for laminated composite plate using Discrete Material Optimization (DMO)

We consider design optimization of vibrating laminated composite plates with the objective of minimizing the total sound power (energy flux) Π radiated from the structural surface S into a surrounding acoustic medium. The same objective has been applied in Du and Olhoff (2007a) for a vibrating isotropic bi-material elastic plate. The present work aims to realize this objective for a laminated composite plate by design optimization of stacking sequence, fiber orientations, and selection of layer materials. This problem is solved by using the so-called Discrete Material Optimization (DMO) approach in the sense that the structural constituents are chosen from among a given set of different candidate materials (Stegmann and Lund, 2005), which may be regarded as an extension of classical topology optimization (see, e.g., Bendsøe and Sigmund, 2003; Rozvany et al., 1992) with a constraint on the total volume of material.

The mathematical formulation of the problem is as follows.

$$\begin{aligned}
&\min_{\mathbf{x}} \left\{ \Pi = \int_S I_n \, dS = \int_S \frac{1}{2} Re(p_f v_n^*) dS \right\} \\
&\text{s.t.} \quad (\mathbf{K} - \omega_p^2 \mathbf{M})\mathbf{U} = \mathbf{P} + \mathbf{L}\mathbf{P}_f \\
&\qquad \mathbf{C}_\alpha \mathbf{P}_f = \mathbf{G}\mathbf{U} - \mathbf{H}\mathbf{P}_f \\
&\qquad \sum_{e=1}^{N^e} \left(\sum_{l=1}^{N^l} \sum_{i=1}^{n^l} (\xi_{eli} u_i) t_l \right) A_e \leqslant \overline{R} \\
&\qquad 0 < x_{\min} \leqslant x_j \leqslant x_{\max} < 1, \quad j = 1, N^{dv}
\end{aligned} \tag{1}$$