International Journal of Solids and Structures 46 (2009) 3873–3881

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Dynamics of a system comprising an orthotropic layer and orthotropic half-plane under the action of an oscillating moving load

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ARTICLE INFO

Article history:
Received 4 December 2008
Received in revised form 15 July 2009
Available online 24 July 2009

Keywords:
Orthotropic material
Critical velocity
Dynamic response
Covering layer
Half-plane
Oscillating moving load

ABSTRACT

This paper investigates the dynamic response to a time-harmonic oscillating moving load of a system comprising a covering layer and half-plane, within the scope of the piecewise-homogeneous body model utilizing of the exact equations of the linear theory of elastodynamics. It is assumed that the materials of the layer and half-plane are anisotropic (orthotropic), and that the velocity of the line-located time-harmonic oscillating moving load is constant as it acts on the free face of the covering layer. Our investigations were carried out for a two-dimensional problem (plane-strain state) under subsonic velocity for a moving load in complete and incomplete contact conditions. The corresponding numerical results were obtained for the stiffer layer and soft half-plane system in which the modulus of elasticity of the covering layer material (for the moving direction of the load) is greater than that of the half-plane material. Numerical results are presented and discussed for the critical velocity, displacement and stress distribution for various values of the problem parameters. In particular, it is established that the critical velocity of the moving load is controlled mainly with a Rayleigh wave speed of a half-plane material and the existence of the oscillation of the moving load causes two types of critical velocity to appear: one of which is less, but the other one is greater than that attained for the case where the mentioned oscillation is absent.

1. Introduction

In this paper, within the scope of the piecewise-homogeneous body model and through the employment of the exact equations of the linear theory of elastic waves the dynamics of a system comprising an orthotropic layer and orthotropic half-plane under the action of an oscillating moving load is studied. Note that this study is a continuation of that which was made in the paper by Akbarov and Ilhan (2008) for the case where the external moving load is not oscillating. At the same time, in that paper the brief review of the related investigations carried out by Achenbach et al. (1967), Auersch (2006), Karlström, 2006, Bespalova (2007), Madshus and Kaynia (2000), Green et al. (1952), Biot (1965), Truestell and Noll (1965), Guz (1986a,b, 2004), Akbarov and Ozisik (2004), Zhuk and Guz (2007), Akbarov (2006, 2007a), Yahnioglu (2007) and many others are given. Moreover in that paper the moving load problems are classified from various points of view, one of which is based on the investigative approach.

As a more detailed description and analysis of the classifications mentioned above and specific approaches were given in the paper by Akbarov and Ilhan (2008), therefore we will not dwell on those here. Nevertheless we will cover herein those arising after the writing of that paper and, where necessary, those that must be considered again.

In Babich et al. (1986), the dynamical response was considered for a system consisting of a layer and pre-strained half-plane. The equation of motion for the covering layer was described by Timoshenko beam theory, but the equation of motion for the half plane was described by three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB). The solution to the corresponding boundary value problem was determined by using the exponential Fourier integral transformation. Corresponding numerical investigations were made for the case where constitutive relations for the half-plane material were described in terms of harmonic potential. Moreover, it was assumed that the speed of the moving load was constant and the subsonic case had been taken into consideration. These numerical investigations led to the further study of the parameters' influence on critical velocity in the second study by Babich et al. (1988) utilizing the complex potentials of TLTEWISB. In Babich et al. (2008a,b) the foregoing investigations of these authors are developed for the supersonic moving load acting for compressible (2008a) and incompressible (2008b) bodies.

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doi:10.1016/j.ijsolstr.2009.07.012

Akbarov et al. (2007) employed the findings of Babich et al. (1986, 1988) in developing a case where the covering layer is also strained initially, and where the equation of motion for this layer is also described by TLTEWISB; from this, the influence of the problem parameters on the critical velocity was studied. However, Akbarov et al. (2007) assumed the materials of the covering and half-plane to be isotropic. This assumption significantly restricts the theoretical investigations in terms of controlling the critical velocity values for the moving load and the stresses acting on the interface plane through the mechanical properties of the layer and half-plane materials. Therefore, the study by Akbarov and Ilhan (2008) further develops the investigation in Akbarov et al. (2007) for the case where the materials of the covering layer and half-plane are anisotropic (orthotropic). Note that the anisotropy of the covering layer materials in the aforementioned systems may occur as a result of oriented reinforcing elements present in these materials. At the same time, under certain conditions, multi-layered soil (half-plane) or a multi-layered covering plate can be modeled as a homogeneous orthotropic material with effective mechanical properties.

Furthermore, it should be noted that the foregoing determination of a system consisting of a stiffer layer and soft half-space or soft layer and stiffer half-space is not applicable in cases where the materials of the covering layer or of the half space are orthotropic, because, in the latter case, there are six independent moduli of elasticity and many wave speeds associated with these moduli. Therefore, for cases where the materials of the covering layer or of the half-space are orthotropic, whether the system consists of a "stiffer layer and soft half-space" or "soft layer and stiffer half-space" must be indicated. In the aforementioned paper by Akbarov and Ilhan (2008), within the "stiffer layer and soft half-space" system, the modulus of elasticity of the covering layer material in the moving load direction is greater than that of the half-plane material, and the corresponding numerical results are presented for this case. These results illustrate the influence of the mechanical properties of the covering layer material and the influence of the initial stresses on the values of the moving load's critical velocity and on the stresses acting on the interface plane between the layer and half-plane.

As noted by Hussein and Hunt (2007), Degrande and Schillemans (2001), Lefeuve-Mesgouez et al. (2000), Auersch (2008) and many others, in reality high-speed trains, cars and other high-speed transportation vehicles modeled as moving loads are accompanied by their own oscillations. To determine how these accompanying oscillations act on the dynamical response of the system considered requires corresponding additional investigations. These investigations are the subject of the present paper; in other words, in the present paper the investigations carried out in Akbarov and Ilhan (2008) are developed for time-harmonic oscillating moving loads. However, in order to focus on the study of the influence of the aforementioned oscillation of the moving load on the dynamics of the system considered in the present paper (in contrast with the paper by Akbarov and Ilhan (2008)) it is assumed that there are not any initial stresses in the constituents of this system.

Throughout the paper repeated indices indicate a summation over their ranges. However, underlined repeated indices are not to be taken as sums.

2. On the formulation of the problem and solution method

The object of the investigation is the same one which is considered in the paper by Akbarov and Ilhan (2008) (with assumption $\sigma_{11}^{(k),0} = 0$, where $\sigma_{11}^{(k),0}$ is an initial stress in the kth component of the system) and schematically shown in Fig. 1. At the same time, the notation, the equation of motion, the mechanical and geometrical relations, the contact conditions, the boundary conditions at $x_2 \to -\infty$ and the boundary condition at $x_2 = 0$ with respect to $\sigma_{12}^{(1)}$ which are used and written in the paper by Akbarov and Ilhan (2008) occur also in the present paper and therefore for reducing the size of the paper we do not rewrite they here again. However the boundary condition with respect to $\sigma_{22}^{(1)}$ at $x_2 = 0$ is changed and has the following form:

$$\sigma_{22}^{(1)}\Big|_{x_2=0} = -P_0 e^{i\omega t}\delta(x_1 - Vt) \tag{1}$$

By using the coordinate system $x_1' = x_1 - Vt, x_2' = x_2$ which moves with loading force and represents the sought values as $g(x_1', x_2', t) = \bar{g}(x_1', x_2')e^{i\omega t}$ we obtain the following equation from equation of motion for the amplitude of the displacements:

$$\frac{\partial^2 u_1^{(m)}}{\partial x_2^2} + \frac{A_{11}^{(m)}}{\mu_{12}^{(m)}}\frac{\partial^2 u_1^{(m)}}{\partial x_1^2} + \left(1 + \frac{A_{12}^{(m)}}{\mu_{12}^{(m)}}\right)\frac{\partial^2 u_2^{(m)}}{\partial x_1 \partial x_2} = \frac{1}{\left(c_{12}^{(m)}\right)^2}\left(V^2\frac{\partial^2 u_1^{(m)}}{\partial x_1^2} - 2i\omega V\frac{\partial u_1^{(m)}}{\partial x_1} - \omega^2 u_1^{(m)}\right)$$

$$\left(1 + \frac{A_{12}^{(m)}}{\mu_{12}^{(m)}}\right)\frac{\partial^2 u_1^{(m)}}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2^{(m)}}{\partial x_1^2} + \frac{A_{22}^{(m)}}{\mu_{12}^{(m)}}\frac{\partial^2 u_2^{(m)}}{\partial x_2^2} = \frac{1}{\left(c_{12}^{(m)}\right)^2}\left(V^2\frac{\partial^2 u_2^{(m)}}{\partial x_1^2} - 2i\omega V\frac{\partial u_2^{(m)}}{\partial x_1} - \omega^2 u_2^{(m)}\right) \tag{2}$$

where $c_{12}^{(m)} = \sqrt{\mu_{12}^{(m)}/\rho^{(m)}}$.

In Eq. (2), the upper prime in x_1 and x_2, and over bar in $u_1^{(m)}$ and $u_2^{(m)}$ are omitted. In this case, the boundary condition (1) is replaced by the following one:

$$\sigma_{22}^{(1)}\Big|_{x_2=0} = -P_0\delta(x_1) \tag{3}$$

Thus, the other conditions mentioned above are also valid for the new coordinate system $x_1' = x_1 - Vt, x_2' = x_2$ and for the amplitude of the sought values.

Now we consider the solutions to Eq. (2). For this purpose, as in the paper Akbarov and Ilhan (2008) we employ the exponential Fourier transformation with respect to the x_1 coordinate defined as

$$f_F(s, x_2) = \int_{-\infty}^{+\infty} f(x_1, x_2)e^{-isx_1}\,dx_1 \tag{4}$$

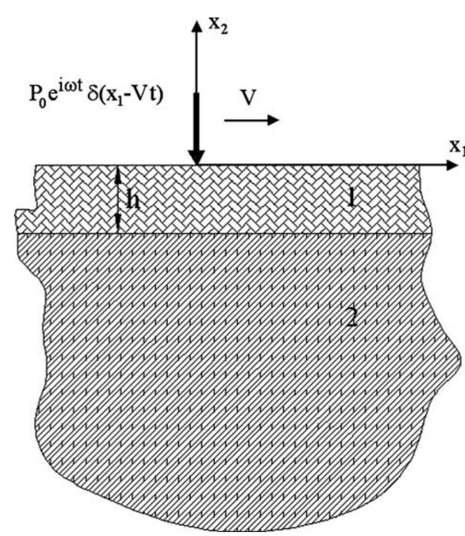

Fig. 1. The geometry of the structure of the half-plane covered by the layer.