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A novel hybrid finite element analysis of inplane singular elastic field around inclusion corners in elastic media

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ABSTRACT

This paper deals with the inplane singular elastic field problems of inclusion corners in elastic media by an *ad hoc* hybrid-stress finite element method. A one-dimensional finite element method-based eigenanalysis is first applied to determine the order of singularity and the angular dependence of the stress and displacement field, which reflects elastic behavior around an inclusion corner. These numerical eigensolutions are subsequently used to develop a super element that simulates the elastic behavior around the inclusion corner. The super element is finally incorporated with standard four-node hybrid-stress elements to constitute an *ad hoc* hybrid-stress finite element method for the analysis of local singular stress fields arising from inclusion corners. The singular stress field is expressed by generalized stress intensity factors defined at the inclusion corner. The *ad hoc* finite element method is used to investigate the problem of a single rectangular or diamond inclusion in isotropic materials under longitudinal tension. Comparison with available numerical results shows the present method is an efficient mesh reducer and yields accurate stress distribution in the near-field region. As applications, the present *ad hoc* finite element method is extended to discuss the inplane singular elastic field problems of a single rectangular or diamond inclusion in anisotropic materials and of two interacting rectangular inclusions in isotropic materials. In the numerical analysis, the generalized stress intensity factors at the inclusion corner are systematically calculated for various material type, stiffness ratio, shape and spacing position of one or two inclusions in a plate subjected to tension and shear loadings.

1. Introduction

Much attention has been paid to inclusion problems by many researchers since the first solution to the ellipsoidal inclusion problems by Eshelby (Eshelby, 1957). The application background is found in microstructures, composite material structures and others (Mura, 1987; Nemat-Nasser and Muneo, 1999; Buryachenko, 2007).

To solve elastic inclusion problems, both the analytical and numerical methods were applied. Among the works on analytical solution, the representative ones that should be mentioned are those of Eshelby (1957), Kröner (1958), Hill (1965), Budiansky (1965), Mori and Tanaka (1973) as well as Kushch et al. (2005). However, these analytical solutions are either limited to very simple geometries such as ellipsoidal inclusions or require high level of mathematical competence. Therefore, most engineering problems of ellipsoidal or irregular shaped inclusions have to resort to numerical methods such as the finite element method (FEM), the volume integral equation method (VIEM), the boundary integral equation method (BIEM) and others.

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The conventional FEM has been used to solve various kinds of inclusions (Ghosh and Mukhopadhyay, 1993; Nakamura and Suresh, 1993 as well as Thomson and Hancock, 1984). However, due to the need for domain discretization, a lot of finite elements must be used for more accurate numerical results. The VIEM is an effective method for the analysis of the inclusions embedded in an isotropic matrix or for coating problems (Buryachenko and Bechel, 2000; Dong and Bonnet, 2002; Dong et al., 2002; Lee et al., 2001 and Nakasone et al., 2000). Compared with the conventional FEM, the VIEM only discretizes the inclusion parts. However, it is difficult to extend this approach to the anisotropic medium due to the use of complex fundamental solutions. Relative to the conventional FEM and VIEM, the BIEM is a more efficient and accurate one for solving inclusion problems. It has been successfully applied for the solution of inclusion problems of various shapes (Tan et al., 1992; Chen and Nisitani, 1993; Chen, 1994; Noda et al., 2000; Dong et al., 2003). However, the method also needs the fundamental solution, which is not easy-to-obtain or impossible-to-obtain for composite materials; and the boundary integral equation must be formulated for each inclusion, which might be inconvenient for solving problems containing many inclusions of irregular shapes. In comparison with the BIEM in the accuracy of numerical solutions, the hybrid-stress finite element method developed more than 40 years ago by Pian (1964) is now well recognized as a powerful and easy-to-use tool for solving a variety of two-dimensional linear elasticity problems containing a single or multiple singular points (Tong et al., 1973; Tong, 1977; Moriya, 1984; Lee and Gao, 1995; Zhang and Katusbe, 1995, 1997; Wang et al., 2004 and the authors, 2001a, 2001b, 2007a, 2007b, 2008). This makes the method attractive and potentially very useful in micromechanics of fibrous composites because it provides an efficient tool for analyzing the advanced one-inclusion or many-inclusion model problems with an accurate account for elastic behavior in the near-field region. In the numerical study of composite materials reinforced by circular inclusions, the method has been applied successfully only by Zhang and Katusbe (1995, 1997).

At the same time, to the best of the author's knowledge, the studies related to interacting non-ellipsoidal inclusions by the hybrid-stress finite element method are absent. Moreover, a numerical solution of even a single non-ellipsoidal inclusion by the hybrid-stress finite element method could not be found in the literature. This may result from a fact that constructing a super inclusion corner element needs the analytical solution of the singularity order and angular variation of singular elastic field that reflect local elastic behavior around the inclusion corner, whose derivation introduces formidable mathematical difficulties for most composite materials.

In the present work, a one-dimensional finite element formulation developed by Sze and Wang (2000), Sze et al. (2001) is first applied for the numerical solution of the order of stress singularities and the angular variation of stress and displacements fields. These numerical fields are subsequently used to develop a super element that simulates the elastic behavior around an inclusion corner. The super element is finally incorporated with standard four-node hybrid-stress elements to constitute an *ad hoc* hybrid-stress finite element method for the analysis of local singular stress fields arising from inclusion corners. To compare the available reference solutions, the *ad hoc* finite element method is used to solve the problem of a single rectangular or diamond inclusion in isotropic materials under longitudinal tension. To present its applicability, the present *ad hoc* finite element method is extended to discuss the inplane singular elastic field problems of a single rectangular inclusion in an anisotropic plate and of two interacting rectangular inclusions in an isotropic plate. In the numerical analysis, the generalized stress intensity factors at the inclusion corner are systematically calculated for various material types, stiffness ratio, shape and spacing position of one and two inclusions in a plate subjected to tension and shear loadings.

2. Expressions for total elastic fields around an inclusion corner

Consider an infinite plate containing an irregular-shaped inclusion, as shown in Fig. 1. The singular stress field around the inclusion corner is analyzed. It is convenient to use a local coordinate system centered at the corner to analyze the local elastic behavior. Fig. 2 shows the local configuration around the corner o. In Fig. 2, two dissimilar wedges with subtending angles of α and $\alpha_1 + \alpha_2$ ($\alpha + \alpha_1 + \alpha_2 = 2\pi$), respectively, are bonded perfectly along both of their interfaces. Each of which may be made of an isotropic, anisotropic, piezoelectric or else material. Wedge 1 is occupied by domain Ω^1, and wedge 2 by domain Ω^2. Let (r, θ) be a local polar coordinate system centered at the corner o, so that the axis of $\theta = 0$ is the bisector of the two wedges. Therefore, when setting $\theta = 0$, we have $\alpha_1 = \alpha_2$. The stress and displacement field near the corner is evaluated by using a one-dimensional eigenanalysis finite element formulation. The formulation is straightforward as will be shown herein.

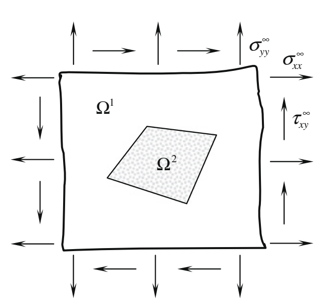

Fig. 1. An infinite plate containing a non-ellipsoidal inclusion.