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An exact theory of interfacial debonding in layered elastic composites

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ABSTRACT

An exact theory of interfacial debonding is developed for a layered composite system consisting of distinct linear elastic slabs separated by nonlinear, nonuniform decohesive interfaces. Loading of the top and bottom external surfaces is defined pointwise while loading of the side surfaces is prescribed in the form of resultants. The work is motivated by the desire to develop a general tool to analyze the detailed features of debonding along uniform and nonuniform straight interfaces in slab systems subject to general loading. The methodology allows for the investigation of both solitary defect as well as multiple defect interaction problems. Interfacial integral equations, governing the normal and tangential displacement jump components at an interface of a slab system are developed from the Fourier series solution for the single slab subject to arbitrary loading on its surfaces. Interfaces are characterized by distinct interface force–displacement jump relations with crack-like defects modeled by an interface strength which varies with interface coordinate. Infinitesimal strain equilibrium solutions, which account for rigid body translation and rotation, are sought by eigenfunction expansion of the solution of the governing interfacial integral equations. Applications of the theory to the bilayer problem with a solitary defect or a defect pair, in both peeling and mixed load configurations are presented.

1. Introduction

This paper presents an exact theory of debonding in a plane layered composite system consisting of distinct linear elastic slabs separated by nonlinear decohesive type interfaces of vanishing thickness. The loading on the top and bottom exterior surfaces are arbitrary and defined pointwise while the loading on each of the side surfaces is defined by resultant shear and normal forces and bending moments. The work is motivated by the relatively recent practice of strengthening reinforced concrete slabs by adhering fiber reinforced plastic plates to the surface. Predicting failure in these systems has occupied the attention of the civil engineering community to a considerable degree as evidenced by the numerous papers published on this topic in recent years (e.g., Au and Buyukozturk, 2006; Leung and Tung, 2006; Wang, 2007; Pan and Leung, 2007; Yuan et al., 2007; De Lorenzis and Zavarise, 2008). These analyses are largely concerned with the interface failure problem. Attempts to treat the coupled flexural crack/interface debonding problem have received less attention owing to the obvious complexities surrounding the mechanical modeling of these interactions. Some experimental and modeling work however has been carried out, e.g., Rabinovitch and Frostig (2001), Teng et al. (2003) and Carpinteri et al. (2007). Analyses of the interface failure problem are based on concepts from either fracture mechanics of homogeneous solids and bimaterials or, cohesive zone modeling employing elementary beam theory and/or finite element analysis (FEA). For the former, Hutchinson and Suo (1992) provides a comprehensive review of this work through 1991. The origins of the later approach can be found in an interesting paper by Ungsuwarungsri and Knauss (1987) who employed elementary beam theory and a number of different piecewise linear cohesive force laws in normal mode to analyze fracture along an interface in a double cantilever beam (DCB). In that work the interface is divided into three distinct regions consisting of an unloaded portion (the crack), a "yield" region characterized by a piecewise linear cohesive law and a semi-infinite linear elastic foundation region. Stationary crack and quasi-static crack propagation behavior are presented although the response in each of these circumstances is obtained separately and the transition between the two is not captured. The paper also includes a comparison finite element analysis and discussion on the use of the results for extracting the surface energy and the shape of the cohesive law. Subsequent studies that employed this approach for the bilayer problem consider different geometries and loadings but typically utilize simplified cohesive zone models to capture interface debonding. Rarely are general normal and shear interface force laws employed (an exception to this is a recent paper by Rabinovitch (2008)). Most recently an analysis along the lines of Ungsuwarungsri and Knauss (1987) was carried out for the problem of multiple defects situated on an interface in a DCB configuration (Carpinteri et al., 2008). Like Ungsuwarungsri and Knauss the authors assume the beam is materially uniform and neglect shear allowing the use of an Euler–Bernoulli beam on an

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elastic foundation. Linear spring type interfaces are employed giving rise to material interpenetration when the interface is in compression. The authors complement this work with FEA using a more general interface force law to assess the significance of this deficiency.

A comprehensive finite strain, finite element analysis of debonding of a viscoplastic block from a rigid substrate was carried out by Needleman (1990a,b). This work utilized general nonuniform, nonlinear interface force laws to study quasi-static debonding along a straight interface under uniaxial straining (Needleman, 1990a) and plane strain tension with superimposed hydrostatic stress (Needleman, 1990b). Here, interface nonuniformity was taken in the form of an interface coordinate dependent interface strength, i.e., a nonbonded portion of interface. The use of nonlinear interface force-separation relations to model separation/slip behavior introduces a characteristic force length in the analysis. For decreasing values of this parameter a transition is shown to occur between more or less uniform separation along the bond line to crack-like propagation of the defect. The studies also indicate that interfacial shear leads to shear dominated decohesion when the interface is uniform and there is significant differential lateral contraction between the rigid substrate and the viscoplastic block.

The present paper provides an exact elasticity solution to a composite slab system composed of N distinct layers subject to general loading. As such it is not subject to the usual approximations inherent in beam theory. Furthermore, because the goal is to focus on the detailed features of quasi-static defect propagation our analysis process provides a sharper tool to investigate brittle and ductile interfacial decohesion in solitary and multiple defect interaction problems then the finite element method. In the following section an exact Fourier series solution for a single slab is presented. This sub-problem is similar in some respects to the well-known plane beam problems considered by Timoshenko and Goodier (1970) and, more recently, Soutas-Little (1973) and Barber (2002), but differs in that general, weak boundary conditions are prescribed on the side surfaces. In Section 3, this solution is used to construct integral equations governing interfacial separation for the two-slab (bilayer) and the multi-slab systems fully accounting for rigid body translation and rotation. Section 4 presents solutions for the bilayer, solitary defect[1] problem under peeling load and mixed load while Section 5 treats the defect pair[2] problem for the bilayer. The explicit interface force law used in these applications is primarily of exponential type developed by Xu and Needleman (1993) although any form may be employed (for comparison purposes, in some calculations we utilize the piecewise linear uncoupled normal and shear laws (De Lorenzis and Zavarise, 2008)). In fact, different forms could be employed for different interfaces in systems in which the number of slabs exceeds two. Interface non-uniformity, i.e., defect specification, at any particular interface is captured by a spatially varying interface strength in the form of nonbonded portions of interface. The paper closes with a brief section on conclusions.

2. Solutions for the single slab

Consider a planar linear elastic slab $B = \{(x,y)|x \in (-l,l), y \in (-h,h)\}$ subject to strong (that is pointwise prescribed) boundary conditions on the horizontal surfaces,

$$\begin{aligned} &S_{xy}(x, y=h) = f_x^1(x), \quad S_{yy}(x, y=h) = f_y^1(x) \\ &S_{xy}(x, y=-h) = f_x^2(x), \quad S_{yy}(x, y=-h) = f_y^2(x) \end{aligned} \tag{1}$$

and weak (that is resultant prescribed) boundary conditions on the vertical surfaces,

$$\begin{aligned} &\int_{-h}^{h} S_{xy}(x=l, y)dy = Q_2, \quad \int_{-h}^{h} S_{xy}(x=-l, y)dy = Q_1 \\ &\int_{-h}^{h} S_{xx}(x=l, y)dy = N_2, \quad \int_{-h}^{h} S_{xx}(x=-l, y)dy = N_1 \\ &\int_{-h}^{h} yS_{xx}(x=l, y)dy = M_2, \quad \int_{-h}^{h} yS_{xx}(x=-l, y)dy = M_1 \end{aligned} \tag{2}$$

where (S_{xx}, S_{xy}, S_{yy}) represent the planar components of the stress tensor $\mathbf{S}$ in Cartesian coordinates and $(N_1, Q_1, M_1, N_2, Q_2, M_2)$ are the prescribed axial force, shear force and bending moment per unit depth of the cross-section, respectively (Fig. 1). The loadings $(f_y^1, f_y^2, f_x^1, f_x^2)$, representing normal and shear tractions on the horizontal surfaces, are assumed to be square integreble[3] and consistent with global equilibrium of the slab, but otherwise arbitrary. Note that the superscript on the traction components (f_x^i, f_y^i) indicates top $(i=1)$ or bottom $(i=2)$ surface (for resultants (N_i, Q_i, M_i) the subscript indicates left $(i=1)$ or right $(i=2)$ surface). Depending upon whether the slab surface is interior or exterior the functions $(f_x^1, f_y^1, f_x^2, f_y^2)$ are regarded as applied boundary tractions or, reactive displacement jump dependent interface tractions. The equations of global equilibrium representing force and moment balance are given by,

$$\begin{aligned} &N_2 - N_1 + \int_{-l}^{l} [f_x^1(x) - f_x^2(x)]dx = 0, \\ &Q_2 - Q_1 + \int_{-l}^{l} [f_y^1(x) - f_y^2(x)]dx = 0 \\ &l(Q_2 + Q_1) + (M_1 - M_2) + \int_{-l}^{l} x[f_y^1(x) - f_y^2(x)]dx \\ &\quad - h\int_{-l}^{l} [f_x^1(x) + f_x^2(x)]dx = 0 \end{aligned} \tag{3}$$

where use has been made of the weak boundary conditions (2).

Stress function solutions that satisfy the biharmonic equation $(\Delta\Delta\varphi = 0)$ may be expressed in the form of an eigenfunction expansion,

$$\varphi = \sum_{n=0}^{\infty} \psi_n(y) \cos \alpha_n x + \sum_{n=1}^{\infty} \eta_n(y) \sin \alpha_n x, \quad \alpha_n = \frac{n\pi}{l} \tag{4}$$

with ψ_n, η_n given by,

$$\begin{aligned} &\psi_0(y) = C_{10} + C_{20}y + C_{30}y^2 + C_{40}y^3 \\ &\psi_n(y) = C_{1n} \cosh \alpha_n y + C_{2n} \sinh \alpha_n y + C_{3n} y \cosh \alpha_n y + C_{4n} y \sinh \alpha_n y \\ &\eta_n(y) = D_{1n} \cosh \alpha_n y + D_{2n} \sinh \alpha_n y + D_{3n} y \cosh \alpha_n y + D_{4n} y \sinh \alpha_n y \\ &n = 1, 2, 3, \ldots \end{aligned} \tag{5}$$

The terms $C_{10}, C_{20}y$ are degenerate since they lead to a null stress field, i.e., components

$$S_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \quad S_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \quad S_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} \tag{6}$$

vanish identically. The stress function $C_0xy + C_1x^2$ however leads to Fourier stress components, $S_{xx} = 0$, $S_{yy} = 2C_1$, $S_{xy} = -C_0$. In addition, we must add the stress function $C_2x^2y + C_3xy^2 + C_4xy^3$ which captures axial variation of shear force, normal force and bending moment required by the weak boundary conditions on the surfaces at $x = \pm l$ (Fig. 1). Thus the complete stress function is given by (4) and (5) with $C_{10} + C_{20}y + C_{30}y^2 + C_{40}y^3$ in $\psi_0(y)$ replaced by $C_0xy + C_1x^2 + C_{30}y^2 + C_{40}y^3 + C_2x^2y + C_3xy^2 + C_4xy^3$. The seven constants $\{C_0, C_1, C_2, C_3, C_4, C_{30}, C_{40}\}$ together with the 8 sets of coefficients $\{C_{1n}, C_{2n}, C_{3n}, C_{4n}, D_{1n}, D_{2n}, D_{3n}, D_{4n}, n = 1, 2, \ldots\}$ are

[1] A single nonbonded portion of interface.

[2] Two distinct, separated nonbonded portions of interface.

[3] A function which is piecewise continuous with a finite number of bounded jump discontinuities.