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Nonlocal regularisation of a model based on breakage mechanics for granular materials

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ABSTRACT

We apply a nonlocal regularisation method to a breakage model recently proposed for the constitutive modelling of crushable granular materials. Coupling between material points is taken into account through the nonlocal averaging applied to the yield function. This enables us to introduce a length scale to continuum breakage models, and therefore helps to overcome numerical difficulties arising from physical instabilities encountered when the material effectively undergoes softening due to crushing. The numerical implementation of the nonlocal version of a breakage model is briefly presented. To demonstrate its promising features, this nonlocal model is used in a series of numerical analyses of high pressure shearing processes, in which grain crushing mechanism governs the material behaviour.

1. Introduction

Careful construction of constitutive models that consider essential microscopic mechanisms can explain experimentally observed features of material behaviour through physical arguments. In return, such models are typically advantageous for the numerical analysis of engineering problems, since these may extend to situations that are beyond the experimental phenomenology. The recent theory of breakage mechanics (Einav, 2007a,b) enables to construct constitutive models that account for the microscopic details of grain crushing. Consequently, models of this theory have been applied successfully in the numerical analysis of various geomechanical and geophysical problems where grain crushing plays an important role, e.g. landslides (Nguyen et al., 2008) and high pressure cataclastic shear (Einav and Nguyen, 2009).

Material stability is an issue one should always deal with when developing constitutive models for softening materials. In crushable granular materials, the crushing of particles in confined conditions leads to softening behaviour observed at the macro scale. We assume here that there are no other types of instability caused by inadequate treatments of boundary conditions or initial conditions. Therefore, within the framework of continuum mechanics, material softening from the loss of positive definiteness of the stiffness tensor is the cause of numerical issues that can arise when one tries to analyse a complete boundary value problem (BVP) using local (conventional) constitutive models without a length scale. Mathematically, due to this loss of material stability, the governing differential equations of continuum mechanics will lose its ellipticity (in statics) or hyperbolocity (in dynamics). As a consequence, the continuum modelling of solids/structures made of softening material leads to ill-posed BVPs (Jirásek and Bazant, 2002). In numerical analysis, this ill-posedness leads to the localisation of deformation into a narrow band, the width of which is governed by the resolution of the spatial discretisation. Outside this localisation band, the material is unloaded elastically. The total energy dissipation (i.e., the sum of material dissipation inside the localisation band) therefore reduces with refinement of the spatial discretisation. In the limit, once the element size (e.g. in finite element analysis) approaches zero, the numerical results converge to a solution with zero localisation band size and zero total dissipation. This numerical prediction is physically unrealistic and is due to the inadequacy of conventional continuum mechanics to deal with ill-posed BVPs. The resolution of the above-mentioned problems requires a regularisation through the introduction of a length scale to the material model. In continuum modelling of softening (granular) materials the regularisation can be as simple as possible utilizing the concept of smeared crack/deformation (e.g. Crook et al., 2006) or more sophisticated using micro-polar (Tejchman, 2004a,b; Tejchman and Gorski, 2008) or nonlocal theories (Marcher and Vermeer, 2001; Maier, 2003, 2004; Tejchman, 2004a,b).

Once a particle is broken into smaller fragments, within a granular volume element, its collapse not only induces sudden movements of neighbouring particles, but also leads to the redistribution/transmission of elastically stored and kinetic energies within a certain volume of the material (Nguyen and Einav, 2009a). This highlights the notion of long range interactions between particles, which introduce effects that the classical equilib-

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rium equations of conventional continuum mechanics might not be able to capture. Even without grain crushing effects, numerical analysis of granular materials based on the discrete element method (Ord et al., 2007) also indicated the presence of long range interaction effects, which cannot be captured using conventional continuum modelling. The requirement to take into account the long range interactions in granular materials motivates the development of nonlocal approach to the modelling of crushable granular materials.

In a rational way, an intrinsic length scale of a nonlocal continuum material model should ideally emerge from upscaling micro-structural details. In granular materials, where particle crushing plays the role of the governing micro-structural mechanism, the induced effects should be linked with the length scale that is associated with the long range correlations. In the constitutive modelling, this link is useful in determining parameters of nonlocal model, which involves both the spatial parameters in charge of the interaction between material points and the local parameters that control the pointwise response of a material point. In this study, we avoid entering such an interpretation of the upscaling process; we view the nonlocal regularisation simply as a numerical mean to eliminate unwanted stability issues that existing local constitutive models may encounter. In particular, we adapt a nonlocal regularisation technique and apply it directly to the nonlocal averaging of the yield function of an existing (local) breakage model (Einav, 2007a,b).

The paper is organized as follows. The formulation of the local breakage model will be briefly presented in the next section, following early developments in Einav (2007a,b,c), and focusing on the simplest approach based on linear elasticity. This is then succeeded by the nonlocal regularisation applying to a rearranged form of the breakage/yield criterion, and then the description of a stress return algorithm for nonlocal incremental constitutive equations. We highlight the role of the particular choice of the loading function in the success of the nonlocal regularisation technique. In the case of the breakage model adopted in this study, it is not sufficient to consider as nonlocal only quantities directly controlling the softening process, as suggested in early work on nonlocal damage model by Pijaudier-Cabot and Bazant (1987). The numerical illustrations in the following section demonstrate how nonlocal regularisation can eliminate the mesh-dependence encountered in predictions by local models. We note that these results are only intended to show numerically the effectiveness of the regularisation. More rigorous localisation analysis (e.g. the dispersion analysis by Pijaudier-Cabot and Benallal (1993), Sluys et al. (1993), Borino et al. (2003) is to be covered in a forthcoming paper. In the conclusion, we discuss how the nonlocal regularisation of breakage models can be useful to solving geotechnical/geophysical problems.

2. A local breakage model

We start with the formulation of a local breakage model based on early developments in Einav (2007a,b). We then set up a starting point for introducing a nonlocal regularisation technique which could be applicable to such a model. Standard notations in soil mechanics are used: mean effective stress p, shear stress q, total volumetric strain ε_v and elastic volumetric strain ε_v^e, total shear strain ε_s and elastic shear strain ε_s^e. The energy potential Ψ and dissipation potential Φ are assumed to take the following forms (Einav, 2007c):

$$\Psi = \frac{1}{2}(1-\vartheta B)\left(K\varepsilon_v^{e2} + 3G\varepsilon_s^{e2}\right) \quad (1)$$

$$\Phi = \sqrt{\Phi_B^2 + \Phi_p^{v2} + \Phi_p^{s2}} \quad (2)$$

in which B is the scalar breakage variable; ϑ is the index property measuring how far the initial *gsd* (grain size distribution) is from the ultimate *gsd* (Einav, 2007a) through second order moments of these distributions; K and G are the bulk and shear modulii, respectively. A more complicated form of the energy potential taking into account the pressure-dependency of the elastic behaviour can be found in Nguyen and Einav (2009a).

In the above we highlight that the Helmholtz free energy potential is specifically formulated in terms of elastic strains and not in terms of the total strain minus the plastic strain. This is consistent with the work of Rubin (2001), who motivates the physical reasons why "constitutive equations must depend on state variables that, in principle, can be measured without any prior knowledge of the past history of deformation of the material. Within the context of this notion of state, elastic strain is a state variable, whereas the total strain and plastic strains are not state variables since they are measured with respect to an arbitrary reference configuration". In the line of this proposition, the dissipation potential can only be a function of the rate of plastic deformation rather than its cumulative value. Furthermore, the proposition above does not require the acceptance of the assumption on the decomposition of total strain into plastic and elastic strains in additive form. The notion of elastic–plastic decomposition can only be valid in its incremental form (see Collins and Einav, 2005). Similar to the damage mechanics theory of solid-like materials the breakage mechanics theory of brittle granular materials also deals with permanent micro-structural fabric alterations, and creation of new surface area. However, these theories are fundamentally different, as they deal with totally distinguishable materials. The mathematical differences initiate from the dissimilar definition of the internal variables, i.e., breakage B in breakage mechanics (Einav, 2007a) and damage D in damage mechanics (e.g. Lemaitre, 1992). The breakage, B, weighs the relative distance of a current *gsd* $g(x)$ from an initial and ultimate *gsd*'s $g_0(x)$ and $g_u(x)$, via the relation:

$$g(x) = g_0(x)(1-B) + g_u(x)B \quad (3)$$

On the other hand, in damage mechanics, the effect of micro-voids or micro-crack openings on the solid behaviour is accounted for through a directional damage variable $D = A_v/A$, where A_v is the void area and A the total cross sectional area with normal vector $\mathbf{n}$ of a representative volume element (RVE). Furthermore, while the general mathematical structure of the Helmholtz energy potentials in both theories may seem related, these are derived from entirely different physical arguments. In particular, in the simplest form of damage mechanics, where a uniform distribution of voids/micro-cracks within a RVE is assumed, D becomes a scalar variable, and the corresponding Helmholtz free energy includes the multiplication of the term $(1-D)$ by the potential energy of an 'un-damaged' continuum. This structure is the outcome of the strain equivalence principle (Lemaitre, 1971). On the other hand, in breakage mechanics, the term $(1-\vartheta B)$ appears instead of the $(1-D)$ term and multiplies the strain energy in a reference particle size. The term $(1-\vartheta B)$ emerges from a statistical homogenization operation via the *gsd* of Eq. (3), and the universal scaling of the stored energy in the particles with their surface area (Einav, 2007a); the index property ϑ is a signature of this operation.

In Eq. (2), the dissipation potential Φ comprises three parts corresponding to breakage dissipation Φ_B, plastic volumetric dissipation Φ_p^v and plastic shear dissipation Φ_p^s (Einav, 2007b):

$$\Phi_B = \frac{\sqrt{E_B E_c}}{(1-B)\cos\omega}\delta B \quad (4)$$

$$\Phi_p^v = \frac{p}{(1-B)\sin\omega}\sqrt{\frac{E_c}{E_B}}\delta\varepsilon_v^p \quad (5)$$

$$\Phi_p^s = Mp|\delta\varepsilon_s^p| \quad (6)$$