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Local probabilistic homogenization of two-dimensional model foams accounting for micro structural disorder

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ABSTRACT

The objective of the present study is a numerical analysis of disorder effects in solid structural foams caused by their random irregular micro structure. Using a strain energy based concept, the effective material response is computed in a geometrically non-linear homogenization analysis. The probabilistic homogenization is based on the analysis of a large scale statistically representative volume element. The stochastic information about the scatter in the material response on the lowest possible level is generated by a subsequent division of the representative volume element in substructures consisting of a single cell wall intersection and parts of the adjacent cell walls. For each of the substructures, a homogenization analysis is performed. The results for the local effective stress and strain components are evaluated by means of stochastic methods. The approach is illustrated by a number of examplary studies on the uncertainty of the effective material response of two-dimensional model foams with linear and non-linear elastic material behavior on the cell wall level of structural hierarchy.

1. Introduction

Solid structural foams are important materials in modern lightweight construction which are used as core materials for sandwich structures as well as in other applications. The main advantage of structural foams is their low specific weight at reasonable stiffness and strength levels. The classical field for application of lightweight principles is the aerospace industry. Nevertheless, in the recent time, an increasing demand for lightweight structures can be observed in other technological fields, especially in the automotive industry (Ning et al., 2007), railway technology (Kim et al., 2007) or the naval industry (Mouritz et al., 2001) where both, regular honeycomb type cellular structures and micro structurally disordered foams are used. A second important property of solid foams for structural application is their high energy absorption capacity due to their high compressibility. In conjunction with the almost constant stress level during a wide range of compression, this feature makes solid foams a natural choice for all kinds of shock absorbing systems such as crash elements in automotive application or personal protection systems such as safety helmets etc. (Boomsma et al., 2003; Mills et al., 2003).

In the industrial design process, the numerical analysis of structures and components consisting partly or completely of foamed materials is preferrably performed in terms of the macroscopic "effective" properties rather than by detailed models of the given micro structure. The determination of the effective properties can be performed either by experimental measurements or, at least in an assisting manner, by numerical predictions. Theoretical and numerical approaches play an important role for the identification and description of the underlying microscopic deformation and failure mechanisms. Furthermore, theoretical and numerical analyses might reduce the experimental expenses during the design and optimization of materials for specific applications caused by extensive testing.

Since the pioneering studies by Gent and Thomas (1963) as well as by Patel and Finnie (1970) appeared, numerous studies on the effective properties of foams and other low density cellular solids have been published. Most theoretical and numerical studies in this area are based on the analysis of an idealized regular model for the foam micro structure such as the Kelvin's (1887) tetrakaidecahedral foam model used e.g. by Warren and Kraynik (1988) or the cubic foam model proposed by Gibson and Ashby (1982, 1997). Other important regular periodic micro structural models used in the homogenization analysis of solid foams are the pentagonal dodecahedral cell model proposed by Christensen (1987) which, in contrast to the models mentioned previously, has the advantage to yield isotropic results or the more recent four cell model proposed by Weaire and Phelan (1994) which outperforms the Kelvin foam model in terms of Kelvin's (1887) energetic optimality criterion requiring a minimum cell wall surface for a given cell volume.

The main disadvantage of structural foams is their disordered micro structure, leading to a distinct scatter in their effective material properties and thus to a limited reproducability of experimental measurements as well as theoretical predictions of their

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material response. Especially in cases, where the characteristic micro and macro structural length scales do not differ by several orders of magnitude, as e.g. for metal foams with large cell sizes in the range of several millimeters used in sandwich structures with core thicknesses in the range of several centimeters, the probabilistic nature of the foam micro structure and thus the scatter in the effective properties are no longer negligible. In experimental studies by Blazy et al. (2004) as well as by Ramamurty and Paul (2004), it has been shown that uncertainty effects can be observed even for metal foams with less disordered micro structures. In this context, the most important stochastic parameter on the micro scale is the local relative density.

In order to account for the micro structural disorder of solid foams, a number of specialized homogenization approaches has been developed in the recent decade. Most of these studies are based on the computational generation and subsequent homogenization of a representative volume element with random micro structure using different versions of the Voronoï (1908) process. Approaches of this type have been used, among others, by Chen et al. (1999) for two-dimensional model foams featuring different types of imperfections, by Huyse and Maes (2001) or by Roberts and Garboczi (2002) who studied the effective elastic behavior of open cell foams where the occurrence of partially closed cell windows forms an additional micro structural geometric uncertainty. All of the mentioned approaches are based on a single analysis of a large scale representative volume element. Hence, they are capable to the determination of the mean macroscopic material response, whereas the scatter is not accessible. In order to assess the scatter, Shulmeister et al. (1998) as well as Zhu et al. (2000) performed a limited number of repeated executions of the generation and analysis of random computational models for foam micro structures using the Voronoï technique. The latter study also provides a first attempt for a systematic analysis of the effects of micro structural geometric uncertainties on the material uncertainties on the macroscopic level of structural hierarchy.

An alternative route to the use of the Voronoï (1908) procedure in its original and subsidary versions has been proposed by Daxner et al. (2000) as well as by Hohe and Becker (2005), using randomized regular foam models. In the latter study, a large number of repeated executions is employed to generate the stochastic information about the scatter which is assessed in terms of the basic stochastic parameters. As an alternative to the previously mentioned numerical approaches based on random number generation, Fortes and Ashby (1999) have proposed the use of direct probabilistic methods based on the analysis on the probability of the orientation of individual micro structural elements. Similar approaches have been used by Cuitiño and Zheng (2003) using a Taylor averaging technique and by Hall (1993) as well as by Warren and Kraynik (1997) using an orientation averaging approach based on the assumption of equal probability for all orientations of a simplified foam model in space.

For the homogenization analysis of randomly heterogeneous two-phase materials such as particle reinforced composites, a distinct body of literature exists. Among the numerous studies, only a few contributions should be mentioned which are of special interest for the problems the present study is concerned with. Ostoja-Starzewski (2006) has provided a comprehensive study on the size effect of the considered micro structural volume elements concerning different types of particle composites. Special interest is directed to the necessary size of the volume element in order to provide statistically representative results. A similar topic has been analyzed by Kanit et al. (2003) as well as by Lachihab and Sab (2005) concerning different types of particle aggregates. A comprehensive overview over concepts for homogenization of random media is provided in a recent book by Torquato (2002). Nevertheless, in the mechanics of foamed materials, rigorous stochastic methods have not widely been used.

Disadvantage of all approaches in the mechanics of foams mentioned previously is the problem that the scatter of the results and thus the uncertainty of the effective properties can only be assessed if the entire sequence of random generation of a computational model for the micro structure and the subsequent homogenization analysis is executed repeatedly in order to obtain the statistical information. In this context, it has to be noticed that the observed scatter strongly depends on the sample size due to self-averaging effects, resulting in the problem that for large samples the scatter can be predicted from the known scatter observed for small samples, whereas a reverse analysis is impossible. On the other hand, the use of small scale computational models for the micro structure with a limited number of pores might suppress a number of important interaction effects of neighboring cells such as the occurrence of a single large cell surrounded by a large number of cells much smaller in size due to the geometric restrictions.

In order to obtain the scatter on the lowest meaningful level and to circumvent the geometric restrictions imposed by the use of small computational foam models, the present study proposes an alternative approach for the probabilistic homogenization analysis of two-dimensional model foams, similar to the method used by Lachihab and Sab (2005) for particle composites. To avoid the restrictions of small scale models, a large scale, statistically representative volume element is employed. The corresponding random micro structure is generated by means of a Voronoï tesselation in Laguerre geometry (Fan et al., 2004) for which the best performance towards the reliable automatic generation of realistic foam models was observed. Subsequently, a finite element model for the micro structure is generated and its deformation behavior is analyzed under homogeneously distributed prescribed effective strain states. For evaluation purposes, the representative volume element is subdivided into a number of substructures consisting of a single cell wall intersection and half of the adjacent cell walls. For these substructures the effective stress and strain components are determined using a strain energy based homogenization concept. The results are evaluated statistically in terms of the resulting probability distributions for the local effective stress and strain components and the corresponding stochastic parameters.

2. Strain energy based homogenization procedure

The numerical homogenization of the foam micro structures in the present study is performed by means of a general strain energy based concept. Since the procedure has been derived and been employed previously (see Hohe and Becker, 2005), only a brief outline is presented here for a concise presentation.

Consider a mechanical body Ω according to Fig. 1. The body is assumed to be bounded by the external boundary $\partial\Omega^{u} \cup \partial\Omega^{t}$. On $\partial\Omega^{u}$, the displacement components u_i are prescribed whereas the components $\sigma_{ij}n_j = t_i$ of the traction vector t_i are given on $\partial\Omega^{t}$. In addition to the prescribed tractions t_i on the external surfaces, the body Ω might be loaded by distributed body loads f_i. The body is assumed to consist of a material with a cellular micro structure. For an efficient structural analysis on the effective level, the body Ω has to be replaced by a body Ω^{*} with the same shape and dimensions, bounded by the same external boundaries $\partial\Omega^{u}$ and $\partial\Omega^{t}$. The replacement body Ω^{*} has to be subject to similar distributed loads f_i^{*} as well as to similar prescribed surface tractions t_i^{*} and similar prescribed displacements u_i^{*} on the respective portions of the external surfaces as the original body Ω. In contrast to the original body Ω, the artificial replacement body Ω^{*} is assumed to consist of the quasi homogeneous "effective" medium with yet unknown properties.