International Journal of Solids and Structures 46 (2009) 1149–1166

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A constitutive model for magnetostrictive and piezoelectric materials

K. Linnemann [a,*], S. Klinkel [b,*], W. Wagner [b]

[a] Fraunhofer Institut für Kurzzeitdynamik, Ernst-Mach-Instiut, Eckerstraße 4, 79104 Freiburg, Germany
[b] Institut für Baustatik, Universität Karlsruhe (TH), Kaiserstraße 12, 76131 Karlsruhe, Germany

ARTICLE INFO

Article history:
Received 16 July 2008
Received in revised form 13 October 2008
Available online 1 November 2008

Keywords:
Nonlinear constitutive law
Magnetostrictive alloys
Ferroelectric ceramics
Ferromagnetic hysteresis
Smart material
Computational treatment
Finite element method

ABSTRACT

This paper is concerned with a macroscopic nonlinear constitutive law for magnetostrictive alloys and ferroelectric ceramics. It accounts for the hysteresis effects which occur in the considered class of materials. The uniaxial model is thermodynamically motivated and based on the definition of a specific free energy function and a switching criterion. Furthermore, an additive split of the strains and the magnetic or electric field strength into a reversible and an irreversible part is suggested. Analog to plasticity, the irreversible quantities serve as internal variables. A one-to-one-relation between the two internal variables provides conservation of volume for the irreversible strains. The material model is able to approximate the ferromagnetic or ferroelectric hysteresis curves and the related butterfly hysteresis curves. Furthermore, an extended approach for ferrimagnetic behavior which occurs in magnetostrictive materials is presented. A main aspect of the constitutive model is its numerical treatment. The finite element method is employed to solve the coupled field problem. Here the usage of the irreversible field strength permits the application of algorithms of computational inelasticity. The algorithmic consistent tangent moduli are developed in closed form. Hence, quadratic convergence in the iterative solution scheme of governing balance equations is obtained.

1. Introduction

Magnetostrictive alloys and ferroelectric ceramics are smart materials. They have a wide range of application for actuation and sensing, see e.g. Smith (2005). Magnetostrictive materials show an inherent coupling between magnetic field and deformation. This effect bears a resemblance to piezoelectricity which describes a coupling between electric field and deformation. Both materials show similar nonlinear behavior. The purpose of this paper is the development of a constitutive model which accounts for the nonlinear behavior and hysteresis effects of magnetostrictive alloys and ferroelectric ceramics.

Microscopically motivated models approximate the switching processes for each single crystallite. The process is modeled by an energy criterion. The macroscopic behavior is obtained by averaging over a large number of union cells. Microscopically motivated models for ferroelectric ceramics are presented in e.g. Chen and Lynch (2001), Hwang et al. (1995, 1998) and Huber and Fleck (2001). Approaches for magnetostrictive materials are often based on the model of Jiles and Atherton (1986) which was primarily developed for ferromagnetic hysteresis without magnetoelastic coupling. The formulation approximates the bending and movement of the domain walls during the magnetization. Extended models for magnetostrictive material are found in e.g. Sablik and Jiles (1993), Jiles (1995), Calkins et al. (2000), Dapino et al. (2000a,b) and Dapino et al. (2002). With the consideration of switching for each crystallite microscopically motivated models generally lead to a large number of internal variables which increases the numerical effort.

Macroscopical constitutive models are based on a phenomenological description of the material behavior. This reduces the amount of internal variables. A widespread approach is the model of Preisach (1935) which was originally developed to describe the magnetization of ferromagnetics. The choice of parameters allows the simulation of a wide range of hysteresis curves. Formulations for magnetostrictive alloys are proposed in Adly et al. (1991) and Tan and Baras (2004). Applications of the Preisach model for piezoelectric hysteresis are published in Hwang et al. (1995), Pasco and Berry (2004), Yu et al. (2002a,b) and Butz et al. (2005, 2008). The Preisach model is purely phenomenological and not thermodynamically consistent.

Another class of constitutive models are formulated in accordance with the principles of thermodynamics. The material behavior is determined with the definition of a specific free energy function. For piezoelectric ceramics the strains, the polarization, and the temperature often serve as independent variables. For magnetostrictives the magnetization is used instead of the polarization. Here, a common approach are higher order energy

* Corresponding authors.
E-mail addresses: konrad.linnemann@emi.fraunhofer.de (K. Linnemann), sven.klinkel@bs.uka.de (S. Klinkel).

0020-7683/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2008.10.014

functions, see e.g. Carman and Mitrovic (1995), Wan et al. (2003), Zheng and Liu (2005), and Zheng and Sun (2007). Alternatively a definition of the free energy function in sections is used, see Wan et al. (2003) and Smith et al. (2003). The independent variables are splitted in a reversible and an irreversible part for the approximation of the hysteresis behavior. The irreversible quantities serve as internal variables and represent the polarization or magnetization state. The development of the internal variables is determined with the definition of a switching criterion. The model of Fang et al. (2004) for magnetostrictive materials is based on these assumptions. More often, thermodynamically motivated models are applied for piezoelectric ceramics. The approaches of Kamlah (2001), Kamlah and Böhle (2001) and Elhadrouz et al. (2005) use multiple switching criterions to specify begin and end of the irreversible behavior. The multiaxial models of Landis (2002) and McMeeking and Landis (2002) get along with only one switching criterion. The domain processes are restricted by a hardening functions. Schröder and Gross (2004) and Schröder and Romanowski (2005) present a co-ordinate invariant formulation on this assumption. The models of McMeeking and Landis (2002), Kamlah (2001), Schröder and Romanowski (2005) use a one to one relation between irreversible strains and polarization to ensure that domain switching induces irreversible strains. This assumption is sufficient for electrical loading but prevents the simulation of ferroelastic behavior or mechanical depolarization. To circumvent this disadvantage Kamlah (2001) decomposes the irreversible strains in a part for ferroelectric and one for ferroelastic switching processes. A more general approach is published in Landis (2002) and Klinkel (2006a,b). Here coupled switching criterions and hardening functions are used to control the change of the irreversible strains. Mehling et al. (2007) introduce an orientation distribution function as additional internal variable to approximate coupled electromechanical loadings.

In the present paper a thermodynamic motivated constitutive model is developed which accounts for hysteresis effects in ferroelectric ceramics and magnetostrictive alloys. The uniaxial model is embedded in a three dimensional formulation. This benefits an efficient numerical treatment of the model but prevents a reorientation of magnetization or polarization during the simulation. The electric respectively the magnetic field strength is splitted additively. The irreversible field strength serves with the irreversible strains as internal variable. The main aspects of the model may be summarized as follows:

- A thermodynamically consistent constitutive model is presented. The formulation is based on the definition of a free energy function and a switching criterion. A polynomial of second order is used as free energy function. The switching criterion controls domain switching. The center of the switching surface moves in the sense of kinematic hardening in plasticity.
- The strains, the magnetic and the electric field strength respectively are decomposed additively in a reversible and an irreversible part. The split of the field strength is an alternative approach to the decomposition of the magnetization or polarization. Its application is motivated by the numerical treatment of the constitutive model with the finite element method. Here the field strength is described with a scalar potential which serves as nodal degrees of freedom. So no change of variables is necessary. Furthermore, one may draw on algorithms of computational inelasticity.
- A one to one relation between the irreversible quantities is utilized to maintain volume conservation during the irreversible processes. A hardening function restricts the growth of the internal variable near saturation.
- The model is able to reproduce the ferroelectric and butterfly hysteresis of ferroelectric ceramics. Furthermore, the typical hysteresis of magnetostrictive alloys can be simulated. The occurring ferrimagnetic hysteresis behavior of these materials is approximated by an expanded approach which is based on a transformation of the independent variables in a local description. Ferroelastic switching and mechanical depolarization are not considered within this work.
- For the numerical treatment of the problem the finite element method is employed. The constitutive model is implemented in a hexahedral element. Therefore the evolution equation is integrated by an implicit Euler backward algorithm which leads to a local iteration. The algorithmic consistent tangent moduli are formulated in closed form. Accordingly, quadratic convergence in the iterative Newton–Raphson solution scheme is guaranteed.

The outline of the paper is as follows: Section 2 deals with the phenomenological behavior of magnetostrictive and piezoelectric materials. In Section 3 the governing equations of the coupled field problems are introduced. The thermodynamic framework of the constitutive model is presented in Section 4. In Section 5 the algorithmic consistent tangent moduli are developed. An expanded model for ferrimagnetic behavior is presented in Section 6. Section 7 deals with the modeling of ferroelectric ceramics. In Section 8 the variational formulation and the finite element approximation are presented. The numerical examples in Section 9 show the capabilities and main characteristics of the proposed constitutive model.

2. Phenomenological behavior

This section summarizes the behavior of the considered smart materials. At first we account for the origin and phenomenology of magnetostriction. For extended treatises on magnetism of solids, the books of Bozorth (1964) and Engdahl (2000) are recommended. The second part of the section is concerned with piezoelectric coupling, in particular with ferroelectric ceramics. It briefly deals with the differences to the magnetostrictive model. Piezoelectricity is well documented in literature, see Jaffe et al. (1971) and Ikeda (1990). A good survey of the basic phenomenons of ferroelectric ceramics and their nonlinear effects is given in e.g. Kamlah (2001).

2.1. Magnetostrictive materials

The term magnetostriction is applied for several magneto-elastic coupling phenomena. The most utilized effect is the Joule magnetostriction which denotes a deformation of a specimen due to an applied magnetic field $\vec{\boldsymbol{H}}$. The reciprocal Villari effect is characterized by a change of the magnetization $\vec{\boldsymbol{M}}$ induced by a mechanical deformation. Both phenomena occur in all ferro-, ferri, and antiferromagnetic materials. The magnetic properties of these materials originate from magnetic moments $\vec{\boldsymbol{m}}$ which results from the spins of electrons in uncomplete occupied inner orbitals of the atom. Due to spin orbit coupling, which is a purely quantum mechanical interaction, the magnetic moments are closely connected with the shape of the electron hull $-$e. The electrical negative electron hull effects Coulomb forces to the neighboring atoms in the lattice. Regarding the complex shape of $-$e these forces are not isotropic, which is important for magnetostrictive coupling. This correlation is described by means of a sample crystallite depicted in Fig. 1. Above the Curie temperature T_c the magnetic moments are disordered, see Fig. 1a. Cooling the material down below T_c, the exchange interaction causes a parallel ordering of the atoms in the lattice, see Fig. 1b. Applying an external magnetic field the magnetic moments switch in the direction of $\vec{\boldsymbol{H}}$. This process is associ-