International Journal of Solids and Structures 46 (2009) 1667–1675

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

DQM for dynamic response of fluid-saturated visco-elastic porous media ☆

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ARTICLE INFO

Article history:
Received 18 July 2008
Received in revised form 1 November 2008
Available online 24 December 2008

Keywords:
Incompressible fluid-saturated visco-elastic porous media
Porous media theory (PMT)
Dynamic response
Differential quadrature method (DQM)
Differential-algebraic system

ABSTRACT

Based on the Porous Media Theory (PMT), a mathematical model of space-axisymmetrical problems for incompressible fluid-saturated visco-elastic porous media is presented in the case of small deformation, in which the differential-type constitutive relation is applied to describe the mechanical characteristics of solid skeleton. The differential quadrature method (DQM) and the second-order backward difference scheme are used to discretize the governing equations on the spatial and temporal domains, respectively, and a method is proposed to deal with the singularity conditions at points located on the symmetry axis. As application, the dynamic behavior of a column of fluid-saturated elastic porous media is analyzed firstly. The obtained results are compared with the analytical results in the existing literature, they are comparatively accordant, which means that the model and method presented in this paper are correct, and the obtained results are reliable. Further, the dynamic response of a space-axisymmetrical body of fluid-saturated visco-elastic porous media is analyzed, in which the material characteristic of the solid skeleton is described by Burgers model with four parameters.

1. Introduction

With the development of science and technology, the investigation of porous media plays an important role in its classical application fields, such as soil mechanics, hydrology, etc., furthermore, it becomes the key to development of emerging sciences and technologies as well, such as, fluid–solid coupling problems in exploitation of the oil and natural gas, mechanical characteristics of the skin, soft tissue and articular cartilage in biology, migration issues of pollutants in the soil media, fluid–solid coupling problems in the rational development and utilization of the geothermal reservoir, etc. Hence, the relevant theoretical analyses and numerical simulation methods are important.

The main theories for porous media include Biot Theory (Biot, 1941, 1956), Porous Media Theory (Bowen, 1982; de Boer, 2000) and Hybrid Mixture Theory (Schrefler et al., 1995; Schrefler, 2002). Based on the Terzaghi's work, Biot first proposed a theory of fluid-saturated porous media. The theory has been successfully applied to many fields of engineering science although it is not based on sound mechanical principles, it introduced quite a good model in an intuitive way (Edelman et al., 2002; de Boer, 2005). At later stage, Boer established a complete porous elastic theory based on the continuous media mixture laws and the concept of volume fraction, in which several micro-mechanical characteristics can be directly applied to describe the macro-mechanical characteristics (de Boer, 2005). At the same time, Boer's model can reflect some effects, such as, dynamical response, material and geometric nonlinearities, etc. easily. Thus, the theory of Boer may be an alternative method to study the mechanical properties of porous media.

Based on the Porous Media Theory (PMT), some analytical and numerical methods have been developed. de Boer et al. (1993) gave the analytical solution for the one-dimensional transient wave propagation in fluid-saturated incompressible elastic porous media by Laplace transform. Breuer (1996, 1999) applied a standard Galerkin finite element method to numerically predict the reflection and refraction of one-dimensional nonlinear longitudinal wave as well as the propagation of two-dimensional linear wave and Rayleigh wave in incompressible fluid-saturated porous media. Under the case of geometrical and material nonlinearity, Diebels and Ehlers (1996) analyzed the dynamic consolidation and seepage problems in fluid-saturated porous media with visco-elastic and elastic-plastic solid skeleton. Based on the assumptions of small deformation and linear elastic material, Aboustit et al. (1985) established variational principles of incompressible static thermal consolidation of fluid-saturated porous media. Based on the similar assumptions, Yang and Cheng (2003) developed a Gurtin-type variational principle of incompressible fluid-saturated porous media and yielded the corresponding finite element formulas. Yang and He (2003) further extended the variational principles in reference Yang and Cheng (2003), and obtained some relevant generalized variational principles.

☆ This work was sponsored by Shanghai Pujiang Program (07pj14073).
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doi:10.1016/j.ijsolstr.2008.12.006

Most of the problems of the two-phase behavior of a fluid-saturated porous media can often be predicted quantitatively by numerical computation. Only a few analytical solutions are available, which are used to verify the numerical results based on the same theory (Breuer and Jagering, 1999). In this work, based on the Theory of Porous Media (PMT) and the differential quadrature method (DQM), a space-axisymmetrical problem of incompressible isotropic fluid-saturated visco-elastic porous media is analyzed, in which one assumes that the porous media is composed of compressible solid and incompressible fluid. Firstly, governing equations for analyzing space-axisymmetrical problems of incompressible fluid porous visco-elastic media are presented in the case of small deformation, in which the differential-type constitutive relation is applied to describe the characteristic of solid skeleton. Secondly, the DQM is applied to discretize the governing equations on the spatial domain, and yield a set of algebraic-differential equations with respect to time. The second-order backward difference scheme is used to discretize the set of algebraic-differential equations. Two numerical examples are proposed, one of which is to validate the method presented in this paper. In the example, the dynamic response of a column of fluid-saturated elastic porous media is analyzed under the cycle and step loading, and the obtained numerical result is compared with the analytical result (de Boer et al., 1993). One can see that they are comparatively accordant. In another example, the dynamic response of a space-axisymmetrical body in fluid-saturated visco-elastic porous media is studied, in which one assumes that the volume deformation is elastic and the bulk modulus is independent of time, and that the shear deformation obeys Burgers model of visco-elastic material with four parameters.

2. Mathematical model

2.1. Basic field equations of visco-elastic fluid-saturated porous media

Assume that $\boldsymbol{V}$ is the initial spatial domain occupied by incompressible fluid-saturated visco-elastic porous media and $\boldsymbol{S}$ is its surface. Based on the Theory of Porous Media developed by de Boer (2005), a kind of fluid-saturated porous media can be thought as an idealization mixture, and it is uniformly composed of solid and fluid phases ϕ^α($\alpha = S$: solid skeleton; $\alpha = F$: fluid phase), and each of them owns its individual motion. Thus, we have

$$\boldsymbol{x}^\alpha = \boldsymbol{x}^\alpha(\boldsymbol{X}, \boldsymbol{t}), \quad (\boldsymbol{X}, \boldsymbol{t}) \in \boldsymbol{V} \times [0, +\infty] \tag{1}$$

Based on the concept of volume fraction, if neglecting the mass and energy exchanges between the solid skeleton with the fluid, we have the field equations of incompressible fluid-saturated porous media.

The volume fractions satisfy

$$\boldsymbol{n}^S + \boldsymbol{n}^F = 1 \tag{2}$$

in which, $n^\alpha(\alpha = S,F)$ is the volume fraction of the constituent ϕ^α.

Balance equation of mass is

$$(\dot{\rho}^\alpha)_\alpha + \rho^\alpha \operatorname{div} \dot{\boldsymbol{x}}^\alpha = \boldsymbol{0}, \quad \alpha = S, F \tag{3a}$$

in which, ρ^α is the partial density of ϕ^α, and $\rho^\alpha = n^\alpha \rho^{\alpha R}$, here, $\rho^{\alpha R}$ is the real density, the symbol $(\bullet)_\alpha = \frac{\partial(\bullet)}{\partial t} + grad(\bullet) \cdot \boldsymbol{x}'_\alpha$ denotes the material time derivatives, $\boldsymbol{x}'_\alpha$ characterizes the constituent velocity of ϕ^α.

Balance equations of momentum and momentum moment are

$$\operatorname{div} \boldsymbol{T}^\alpha + \rho^\alpha(\boldsymbol{b}^\alpha - \ddot{\boldsymbol{x}}_\alpha) + \hat{\boldsymbol{p}}^\alpha = \boldsymbol{0}, \quad \boldsymbol{T}^\alpha = \boldsymbol{T}^{\alpha T}, \quad \alpha = S, F \tag{4}$$

Here, $\boldsymbol{T}^\alpha$ is the Cauchy stress tensor, $\boldsymbol{b}^\alpha$ the body force density, $\hat{\boldsymbol{p}}^\alpha$ is the interaction between the solid skeleton with the fluid phase and satisfies the relation: $\hat{\boldsymbol{p}}^S + \hat{\boldsymbol{p}}^F = 0$. And also, the superscript T denotes transpose.

Based on PMT, for the linear isotropic visco-elastic solid skeleton and nonviscous fluid, the stress tensor $\boldsymbol{T}^\alpha$ and the force $\hat{\boldsymbol{p}}^\alpha$, as a consequence of incompressibility ($\rho^{\alpha R} = const$), can be expressed as

$$\boldsymbol{T}^S = -n^S p \boldsymbol{I} + \boldsymbol{T}^{SE}, \ \boldsymbol{T}^F = -n^F p \, \boldsymbol{I} + \boldsymbol{T}^{FE}, \ \hat{\boldsymbol{p}}^F = p \operatorname{grad} n^F + \hat{\boldsymbol{p}}^{FE} \tag{5a}$$

$$\boldsymbol{T}^{FE} = 0, \ \hat{\boldsymbol{p}}^{FE} = -S_v(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) \tag{5b}$$

in which, $\boldsymbol{u}^\alpha = \boldsymbol{x}^\alpha - \boldsymbol{X}^\alpha(\alpha = S,F)$ are the displacements of solid skeleton and fluid phase, respectively, $\boldsymbol{T}^S$ and $\boldsymbol{T}^F$ are the total stress tensors of solid skeleton and fluid phase, respectively, $\boldsymbol{T}^{SE}$ and $\boldsymbol{T}^{FE}$ are the corresponding extra Cauchy stress tensors, respectively, p is the effective pore pressure of incompressible fluid phase. Set effective specific weight of fluid is $\gamma^{FR} = \rho^{FR}|b|$, the coupled interaction between the solid skeleton with the fluid phase can be described by $S_v = \frac{(n^F)^2 \gamma^{FR}}{\kappa^F}$, in which, κ^F is Darcy permeability coefficient.

The volume fraction of solid skeleton n^S can be denoted as $n^S = n^S_{0S}(\det(x_{i,K}))^{-1} \approx n^S_{0S}$ in the case of small deformation. Namely, the volume fraction n^Scan be approximated by n^S_{0S}, and n^S_{0S} is the solid volume fraction in the initial configuration.

When a differential-type visco-elastic constitutive relation is applied to describe the material characteristics of solid skeleton, the constitutive equation of solid skeleton can be expressed as

$$P' \boldsymbol{S}^{SE} = Q' \boldsymbol{e}^S, \quad P'' \boldsymbol{I}^{SE}_T = Q'' \boldsymbol{I}^S_\varepsilon \tag{6}$$

in which, $\boldsymbol{S}^{SE}$ and $\boldsymbol{e}^S$ are the effective stress and strain deviators, respectively, $\boldsymbol{I}^{SE}_T = \boldsymbol{T}^{SE}_{ii}, \boldsymbol{I}^S_\varepsilon = \varepsilon^S_{ii}$ are the first invariants of Cauchy extra stress tensor $\boldsymbol{T}^{SE}$ and strain tensor $\boldsymbol{\varepsilon}^S$, P', Q' and P'', Q'' are differential operators describing the viscous characteristics of material, and given as: $P' = \sum p'_i \frac{d^i}{dt^i}, Q' = \sum q'_i \frac{d^i}{dt^i}, P'' = \sum p''_i \frac{d^i}{dt^i}, Q'' = \sum q''_i \frac{d^i}{dt^i}$, here, p'_i, q'_i, p''_i, q''_i are material parameters. When $P' = 1$, $Q' = 2G$ and $P'' = 1$, $Q'' = 3K$, Eq. (6) gives the elastic constitutive equation of solid skeleton. In addition, the relations between the effective stress $\boldsymbol{S}^{SE}$ and strain deviator $\boldsymbol{e}^S$ with the Cauchy extra stress tensor $\boldsymbol{T}^{SE}$ and strain tensor $\boldsymbol{\varepsilon}^S$ are given as

$$\boldsymbol{T}^{SE} = \boldsymbol{S}^{SE} + \frac{1}{3} \boldsymbol{I}^{SE}_T, \quad \boldsymbol{\varepsilon}^S = \boldsymbol{e}^S + \frac{1}{3} \boldsymbol{I}^S_\varepsilon \tag{7}$$

In the case of small deformation, the linearized Langrangian strain tensor is defined as

$$\boldsymbol{E}^S = \frac{1}{2}(\operatorname{grad} \mathbf{u}^S + \operatorname{grad}^T \mathbf{u}^S) \tag{8}$$

2.2. Boundary and initial conditions

To solve the problem, it is necessary to give the boundary conditions of solid skeleton and fluid phase. For the solid skeleton, the boundary conditions of displacement and stress can be given as

$$\boldsymbol{u}^S = \overline{\boldsymbol{u}}^S(\boldsymbol{X}, \boldsymbol{t}), (\boldsymbol{X}, \boldsymbol{t}) \in \boldsymbol{S_u} \times [0, +\infty) \tag{9a}$$

$$\boldsymbol{F} = \overline{\boldsymbol{F}}(\boldsymbol{X}, \boldsymbol{t}), (\boldsymbol{X}, \boldsymbol{t}) \in \boldsymbol{S_T} \times [0, +\infty) \tag{9b}$$

in which, $\boldsymbol{S} = \boldsymbol{S_u} \cup \boldsymbol{S_T}$, $\boldsymbol{F} = \boldsymbol{n} \cdot (\boldsymbol{F}^S + \boldsymbol{F}^F)$, and $\boldsymbol{n}$ is the outside unit normal of the surface $\boldsymbol{S_T}$, $\overline{\boldsymbol{u}}^S$ and $\overline{\boldsymbol{F}}$ are the designated displacement and force vectors on the surface $\boldsymbol{S_u}$ and $\boldsymbol{S_T}$.

For the fluid phase, generally speaking, the porous pressure p or the seepage velocity vector $\boldsymbol{Q}$ is given on the designated boundary, and the boundary conditions can be expressed as

$$p = \overline{p}(\boldsymbol{X}, \boldsymbol{t}), (\boldsymbol{X}, \boldsymbol{t}) \in \boldsymbol{S_p} \times [0, +\infty) \tag{9c}$$

$$\boldsymbol{Q} = \overline{\boldsymbol{Q}}(\boldsymbol{X}, \boldsymbol{t}), (\boldsymbol{X}, \boldsymbol{t}) \in \boldsymbol{S_Q} \times [0, +\infty) \tag{9d}$$

in which, $\boldsymbol{S} = \boldsymbol{S_p} \cup \boldsymbol{S_Q}$, $\quad \boldsymbol{Q} = \boldsymbol{n} \cdot (\boldsymbol{n}^F \boldsymbol{F}^F)$.

One can assume that the initial conditions are