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Static analysis for multi-layered piezoelectric cantilevers

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Abstract

Taking the bonding layers and electrodes into account, the multi-layered piezoelectric cantilevers are studied based on the theory of elasticity. Different from the traditional investigations based on the elementary theory of elasticity, the Airy stress function method is used in the present paper. The stress function and induction function are proposed and determined, and then the exact solutions of the static governing equations are found. The material properties and thickness of different layers may be different in the present investigations. As two special cases, the exact static solutions for both unimorph and bimorph are directly obtained by using the present general solutions. The exact solutions obtained in the present paper are compared with the numerical results and others' investigations, and good agreements are found. In addition, the effects of the properties of both bonding layers and electrodes are discussed. Moreover, the present solution can be used for function graded piezoelectric cantilever beams when the thickness of each bonding layer is taken as zero.

Keywords: Piezoelectric material; FGPM; Multi-layered devices; Actuators; Sensors

1. Introduction

As well-known, multi-layered piezoelectric structures play an important role and are widely used in the engineering. Usually these multi-layered devices can elaborate original transducers such as Dual-frequency or Barker Code transducers or 2-D array elements with relative low electrical impedance, which lead to a greater sensitivity compared with single-layer element (Desmare, 1999). If they are made of piezoelectric polymeric materials, these devices can also offer some advantages over piezoelectric ceramic transducers, including flexibility, ease of preparing large sheets and the ability to undergo large deflections without damage (Marcus, 1984). Because of these advantages and wide applications, multi-layered piezoelectric structures have attracted much attention in recent years. For example, based on the one-dimensional constitutive equations, some simple behaviors of symmetric or non-symmetric piezoelectric cantilever bimorphs were studied (Smits et al., 1991; Smits and Ballato, 1994; Brissaud et al., 2003; Brissaud, 2004). Using the same constitutive equations, impedance and admittance matrices were presented for the analysis of the beam-type piezoelectric

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multi-morph devices (Ha and Kim, 2002). Based on the Bernoulli–Euler beam model, the natural frequencies, maximum displacement and resultant force of a symmetric multi-morph cantilever were obtained (Lee et al., 2005). In another investigation, a new approach for laminated plates with piezoelectric layers was proposed based on a refinement improvement of the electric potential as a function of the thickness coordinate (Fernandes and Pouget, 2002). In this study, the shearing correction of elastic displacement was accounted. For the actuator consisting of a metallic layer covered symmetrically by two transversely isotropic piezoelectric layers, the basic behaviors were analyzed (Lim and He, 2004). For an intelligent beam with single elastic layer and two piezoelectric layers, a static analysis with a voltage applying on these two piezoelectric layers was performed by using Airy stress function method (Lin et al., 2001). In addition, a modeling and optimal design method for piezoelectric microactuators was proposed (DeVoe and Pisano, 1997).

In order to improve the reliability of the sensors or actuators, the functionally graded piezoelectric materials (FGPM) have been proposed and manufactured (Zhu and Meng, 1995). Assuming the piezoelectric coefficient d_{31} as a linear function in thickness direction and keeping other material parameters as constant, the experiment study on the functionally graded piezoelectric actuator showed that with respect to a classical bimorph, the deflection of a functionally graded piezoelectric cantilever actuator is only slightly smaller, whereas the internal mechanical stress is drastically reduced (Hauke et al., 2000). Utilizing the similar assumption but based on the theory of elasticity, some exact solutions for functionally graded piezoelectric sensors and actuators are obtained (Shi, 2002; Liu and Shi, 2004; Chen and Shi, 2005; Shi, 2005). The investigation on a linearly graded flat actuator showed that there is not any stress component in the flat actuator when it is subjected to an external voltage (Liu and Shi, 2004), but it will do in a linearly graded curved actuator (Shi, 2005). Moreover, piezoelectric actuators with functionally graded properties are designed with the aim of maintaining high bending displacement and reducing the stress concentration at the middle interface that exists in standard bimorph actuators (Taya et al., 2003).

Though there is considerable number of papers dealing with multi-layered sensors or actuators, most of the investigations were following Timoshenko's approach or based on the elementary theory of elasticity. On the other hand, the simply supported boundary conditions were taken into account in most previous investigations (Heyliger and Brooks, 1996). Moreover, it is noted that the effects of bonding layers cannot be fully ignored. So, the objective of the current research is to give a precise analysis for the multi-layered piezoelectric sensors or actuators with bonding layers and electrodes based on the theory of elasticity. All the equilibrium conditions and continuous conditions for the stress, displacement and induction as well as electric potential on the interfaces between neighbor layers are exactly satisfied. In the present investigation, a multi-layered cantilever with different material properties and thickness for different layers is studied. It should be noted that the present solution can also be used for analyzing functionally graded piezoelectric cantilever beams by assuming the thickness of each bonding layer to be zero. The organization of the rest of this paper is as follows. The basic equations for piezoelectric materials are summarized in Section 2. The exact solutions for a kind of multi-layered piezoelectric composite cantilevers are obtained in Section 3 and then applied to two special cases in Section 4. In Section 5, some numerical results and comparisons are addressed and good agreements are found.

2. Basic equations

For the piezoelectric actuators, the design theory was presented under the consideration of the effect of bonding layers (Marcus, 1984). This effect on a multi-layered actuator was investigated, which all layers of the actuator were connected in parallel (Shi et al., 2006). Here, we will study a multi-layered actuator which all layers are connected in series. As shown in Fig. 1, the piezoelectric layer and bonding layer are placed alternately. There are $n+1$ elastic layers (including two electrodes and $n-1$ bonding layers) and n piezoelectric layers. Actually, the actuator will become a multi-layered pure piezoelectric actuator when the thickness of each elastic layer is zero, which is a multi-layered model for functionally graded piezoelectric beam. Between the upper and lower surfaces of the actuator there is an external electrical potential V_0. The thickness of the elastic layer k is determined by $(h_{2k-1} - h_{2k-2})$ and the thickness of the piezoelectric layer k is determined by $(h_{2k} - h_{2k-1})$ as shown in Fig. 2. The thickness of both the elastic and the piezoelectric layers may be different. In one implementation of these devices, both the bonding layers are the elastic electrodes. Referring to a