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Stress singularity due to traction discontinuity on an anisotropic elastic half-plane

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Abstract

In a half-plane problem with x_1 paralleling with the straight boundary and x_2 pointing into the medium, the stress components on the boundary whose acting plane is perpendicular to x_1 direction may be denoted by $\boldsymbol{t}_1 = [\sigma_{11}, \sigma_{12}, \sigma_{13}]^T$. Stress components σ_{11} and σ_{13} are of more interests since σ_{12} is completely determined by the boundary conditions. For isotropic materials, it is known that under uniform normal loading σ_{11} is constant in the loaded region and vanishes in the unloaded part. Under uniform shear loading, σ_{11} will have a logarithmic singularity at the end points of shear loading. In this paper, the behavior of the stress components σ_{11} and σ_{13} induced by traction-discontinuity on general anisotropic elastic surfaces is studied. By analyzing the problem of uniform tractions applied on the half-plane boundary over a finite loaded region, exact expressions of the stress components σ_{11} and σ_{13} are obtained which reveal that these components consist of in general a constant term and a logarithmic term in the loaded region, while only a logarithmic term exists in unloaded region. Whether the constant term or the logarithmic term will appear or not completely depends on what values of the elements of matrices $\boldsymbol{\Omega}$ and $\boldsymbol{\Gamma}$ will take for a material under consideration. Elements for both matrices are expressed explicitly in terms of elastic stiffness. Results for monoclinic and orthotropic materials are all deduced. The isotropic material is a special case of the present results.

Keywords: Half-plane; Uniform tractions; Stroh formalism; Anisotropic elasticity; Monoclinic materials

1. Introduction

It is known that the stresses in a linear elastic body will in general possess singularities at points where the geometries of the boundary change abruptly, e.g., those tips of the cracks, or at junctions where different material properties met like those occurred in the free edges of bimaterial problems. There are other points of the

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body where stress singularities may occur, e.g., at points at which concentrated loadings are acting or at points where discontinuous tractions on the surface of the body occurred. Among all these possible causes of stress singularities, the last mentioned above will be our concern for the present analyses. For a half-plane isotropic solid where the straight boundary is subjected to a uniform traction $\boldsymbol{t}_2 = [\tau, \sigma, \hat{\tau}]^{\mathrm{T}}$ (see Fig. 1) with a finite loaded region, the stress components induced on the boundary may be denoted by $\boldsymbol{t}_1 = [\sigma_{11}, \sigma_{12}, \sigma_{13}]^{\mathrm{T}}$. Since σ_{12} is completely determined by the boundary conditions, therefore only the behaviors of the components σ_{11} and σ_{13} are of interests in the following discussions. When a uniform normal stress is applied, the stress σ_{11} contains a constant term on the loaded region the magnitude of which is equal to the applied normal stress. Outside the loaded region, σ_{11} is zero. No logarithmic term will be induced for σ_{11} by the normal loading. This is an exact solution (England, 1971, Muskhelishvili, 1953) which shows that σ_{11} exhibits a discontinuity at the end points of the uniform normal loading. When a uniform in-plane shear loading is applied, there will be instead a logarithmic term being induced for the stress σ_{11} in each loaded and unloaded region. No constant term now exists for σ_{11}. The stress σ_{11} will increase without bound at the end points of the shear loading. For the anti-plane problems, which decouples totally from the in-plane problems for isotropic materials, a logarithmic behavior of σ_{13} is found in both loaded and unloaded regions when the uniform anti-plane shear loading is acting. No constant term is found for σ_{13}.

In this paper, the phenomena of the stress components σ_{11} and σ_{13} due to the traction-discontinuity on the anisotropic elastic surfaces are investigated. These phenomena may be understood by the study of the problem of a half-plane anisotropic elastic solid with uniform tractions applied on the straight boundary. With the solutions to a half-plane solid subjected to a concentrated load on the straight surface (Ting, 1996, Conway and Ithaca, 1953), the problem for uniform traction loadings may be obtained by integrations of the results corresponding to the concentrated loading. The exact expressions for the stresses components σ_{11} and σ_{13} are obtained. It is found that these stress components consist of in general a constant term and a logarithmic term. Constant term appears only in the loaded region and its value is determined by the matrix $\mathbf{\Omega} = \mathbf{N}_3\mathbf{S}\mathbf{L}^{-1} - \mathbf{N}_1^{\mathrm{T}}$. The logarithmic term will exist in both loaded and unloaded regions and its amplitude is determined by the matrix $\mathbf{\Gamma} = \boldsymbol{N}_3\boldsymbol{L}^{-1}$. Whether the constant term or the logarithmic term will appear or not for a material under consideration depends completely on what values of the elements of matrices $\mathbf{\Omega}$ and $\mathbf{\Gamma}$ will take for that material. Explicit expressions of the elements of $\mathbf{\Omega}$ and $\mathbf{\Gamma}$ for general anisotropic materials are expressed in terms of the elastic stiffness with the help of results developed by Liou and Sung (submitted for publication) for matrices $\boldsymbol{L}$ and $\boldsymbol{S}$. The stress components σ_{11} and σ_{13} developed for general anisotropic materials are then specialized to monoclinic materials with plane of symmetry at $x_1 = 0$, $x_2 = 0$ and $x_3 = 0$, respectively, and also to orthotropic materials. By taking special values for elements of $\mathbf{\Omega}$ and $\mathbf{\Gamma}$, the results for isotropic materials may be recovered.

It is noted that present investigations are for the traction boundary condition only. For those problems with displacement prescribed on part of the free surface, readers may refer to the monograph by Muskhelishvili (1953) for isotropic materials and the works done by Hwu and Fan (1998a,b) for general anisotropic mate-

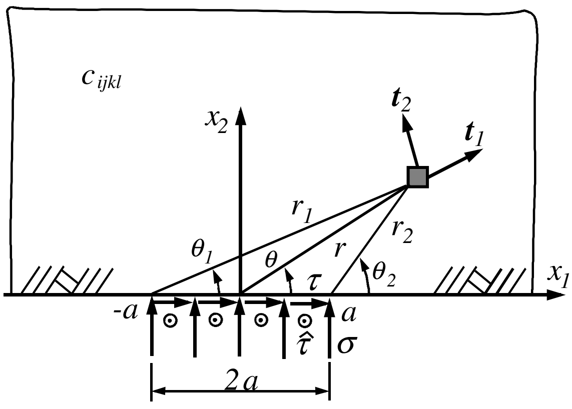

Fig. 1. Geometry of the problem.