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Elastic waves in an electro-microelastic solid

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Abstract

The present study is concerned with the wave propagation in an electro-microelastic solid. The reflection phenomenon of plane elastic waves from a stress free plane boundary of an electro-microelastic solid half-space is studied. The condition and the range of frequency for the existence of elastic waves in an infinite electro-microelastic body are investigated. The constitutive relations and the field equations for an electro-microelastic solid are stemmed from the Eringen's theory of microstretch elasticity with electromagnetic interactions. Amplitude ratios and energy ratios of various reflected waves are presented when an elastic wave is made incident obliquely at the stress free plane boundary of an electro-microelastic solid half-space. It has been verified that there is no dissipation of energy at the boundary surface during reflection. Numerical computations are performed for a specific model to calculate the phase speeds, amplitude ratios and energy ratios, and the results obtained are depicted graphically. The effect of elastic parameter corresponding to micro-stretch is noticed on reflection coefficients, in particular. Results of Parfitt and Eringen [Parfitt, V.R., Eringen, A.C., 1969. Reflection of plane waves from a flat boundary of a micropolar elastic half-space. J. Acoust. Soc. Am. 45, 1258–1272] have also been reduced as a special case from the present formulation.

Keywords: Microstretch; Micropolar; Micro-rotation; Electro-microelastic; Electric; Amplitude; Energy; Reflection

1. Introduction

Eringen (1966) developed the linear theory of micropolar elasticity which is a subclass of the nonlinear theory of simple microelastic solids earlier developed by Eringen and his co-worker (1964a, b) and is a generalization of the classical theory of elasticity. The basic difference between the theory of micropolar elasticity and that of the classical theory of elasticity is the introduction of an independent microrotation vector. Thus, in the theory of micropolar elasticity, the motion in a body is characterized by six degrees of freedom, namely three of translation and three of rotation. The interaction between two parts of a micropolar body is transmitted not only by a force vector but also by a couple resulting in asymmetric force stress tensor and couple stress tensor. Many problems on wave propagation in micropolar elastic media have been investigated by several

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researchers in the past, e.g., Smith (1967), Parfitt and Eringen (1969), Nowacki and Nowacki (1969), Ariman (1972), Tomar and Gogna (1995, 1997) and Tomar et al. (1998) among several others.

The linear theory of microstretch elastic solids developed by Eringen (1990) is a generalization of the theory of micropolar elasticity and is again a subclass of the theory of micromorphic materials. Microstretch elastic solids are those solids in which the material particles can undergo expansion and contraction (stretches), in addition to the translation and rotation. Thus the motion in a microstretch elastic body is characterized by seven degrees of freedom. The transmission of load across a differential element of the surface of a microstretch elastic solid is desribed by a force vector, a couple stress vector and a microstress vector. Composite materials reinforced with chopped elastic fibers and porous materials whose pores are filled with gas may fall in the category of microstretch elastic solids. Tomar and Garg (2005) investigated the possibility of wave propagation and discussed the reflection/transmission phenomena of plane waves at a plane interface between two different microstretch elastic solid half-spaces.

Later, Eringen (2004) extended his theory of microstretch elastic solids to include the electromagnetic interactions and termed it as '*electromagnetic theory of microstretch elasticity*'. These materials include animal bones and nanomaterials. Books by Eringen and Maugin (1990) and Eringen (1999) on microcontinuum and electrodynamic theories are excellent monographs on this pertinent area of research.

Recently, Khurana and Tomar (2007) have explored the possibility of plane waves propagating in an infinite electro-microelastic solid. They found that there exist five plane waves in an infinite electro-microelastic solid namely (i) an independent longitudinal microrotational wave, (ii) two sets of coupled longitudinal waves, which are influenced by the electric effect, and (iii) two sets of coupled transverse waves. All these waves are dispersive. The appearance of the two sets of coupled longitudinal waves having the influence of electric effect is new, which reduces to the longitudinal displacement wave of micropolar elasticity in the absence of electric and microstretch effects. In the present paper, we have derived the condition on propagation of each plane wave existing in an electro-microelastic solid. Reflection phenomenon of each set of these coupled waves (i.e., the coupled longitudinal waves and the coupled transverse waves) at a stress free flat surface of an electro-microelastic solid half-space is investigated. The amplitude ratios and the energy ratios of various reflected waves are presented and computed numerically for a specific model. We have also compared the square of phase speeds of various waves at different limiting values of frequency. The square of the phase speeds of various existing waves are computed and their variations are depicted graphically against the frequency ratio. The variations of the square of speeds of both sets of coupled longitudinal waves with frequency ratio is shown for three different cases, namely, $\lambda_2^2 >=< \alpha_0(1+\chi^E)$. The variations of modulus of amplitude and energy ratios against the angle of incidence are computed and shown graphically. The behavior of amplitude ratios with the frequency ratio is also depicted graphically. It is found that their is no dissipation of energy at the boundary surface during reflection. The problem of Parfitt and Eringen (1969) has been reduced as a limiting case of the present formulation.

2. Basic equations and constitutive relations

In the absence of body force, body couple and body microstretch force densities, the field equations in a linear isotropic and homogeneous electro-microelastic solid medium when the current vector **J**, volume charge density q_e, magnetic flux vector **B** and thermal effect T are ignored, reduce to (Eringen, 2004)

$$(c_1^2+c_3^2)\nabla\nabla\cdot\mathbf{u}-(c_2^2+c_3^2)\nabla\times\nabla\times\mathbf{u}+c_3^2\nabla\times\mathbf{\Phi}+\bar{\lambda}_0\nabla\psi=\ddot{\mathbf{u}}, \tag{1}$$

$$(c_4^2+c_5^2)\nabla\nabla\cdot\mathbf{\Phi}-c_4^2\nabla\times\nabla\times\mathbf{\Phi}+\omega_0^2\nabla\times\mathbf{u}-2\omega_0^2\mathbf{\Phi}=\ddot{\mathbf{\Phi}}, \tag{2}$$

$$c_6^2\nabla^2\psi-c_7^2\psi-c_8^2\nabla\cdot\mathbf{u}+c_9^2\nabla\cdot\mathbf{E}=\ddot{\psi}, \tag{3}$$

$$(1+\chi^E)\nabla\cdot\mathbf{E}+\lambda_2\nabla^2\psi=0, \tag{4}$$

$$\nabla\times\mathbf{E}=0, \tag{5}$$

where $c_1^2=(\lambda+2\mu)/\rho$, $c_2^2=\mu/\rho$, $c_3^2=K/\rho$, $c_4^2=\gamma/\rho j$, $c_5^2=(\alpha+\beta)/\rho j$, $c_6^2=2\alpha_0/\rho j_0$, $c_7^2=2\lambda_1/3\rho j_0$, $c_8^2=2\lambda_0/3\rho j_0$, $c_9^2=2\lambda_2/\rho j_0$, $\omega_0^2=c_3^2/j$, $\bar{\lambda}_0=\lambda_0/\rho$, λ and μ are Lame's constants; K, α, β and γ are micropolar constants; λ_0, λ_1 and α_0 are microstretch constants; ρ is the density of the medium, j and j_0 are constants, ψ is