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A shear-lag model for three-dimensional, unidirectional multilayered structures

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Abstract

A shear-lag model is derived for unidirectional multilayered structures whose constituents vary throughout the cross-section through the extension of an existing optimal shear-lag model suitable for two-dimensional planar structures. Solution algorithms for a variety of boundary conditions are discussed. Numerical predictions for a single-fiber composite and a unidirectional laminated composite are presented. Comparison of the predicted interfacial shear stresses and average normal stresses to finite element analysis demonstrates that this shear-lag model can be used to rapidly estimate the average normal stress distribution in the various constituents, although the interfacial shear stresses are less accurate. Possible applications and limitations of the new model are finally discussed.

Keywords: Shear-lag; Laminated composite; Multilayered structure

1. Introduction

The shear-lag method has been widely applied as a computationally efficient analytical method to analyze the stress distribution in fiber reinforced composite structures (Cox, 1952; Hedgepeth, 1961). Nairn and Mendels (2001) present an extensive review of the literature on shear-lag methods applied to axisymmetric and two-dimensional (2D) planar geometries. Historically, many researchers have combined the shear-lag based calculation of load transfer from broken fibers to surrounding intact fibers with statistical fiber failure models for the prediction of composite strengths (Landis et al., 1999; Landis and McMeeking, 1999; Beyerlein and Landis, 1999; Okabe et al., 2001; Okabe and Takeda, 2002; Xia et al., 2002). More recently, the shear-lag method has been applied to estimate axial stresses in embedded optical fiber sensors for smart structures (Yuan and Zhou, 1998; Ansari and Yuan, 1998; Yuan et al., 2001; Okabe et al., 2002; Prabhugoud and Peters, 2003; Li et al., 2006). The goal of this article is to extend the shear-lag method to multilayer unidirectional structures where the geometry varies throughout the cross-section in a non-periodic manner. Fig. 1 shows

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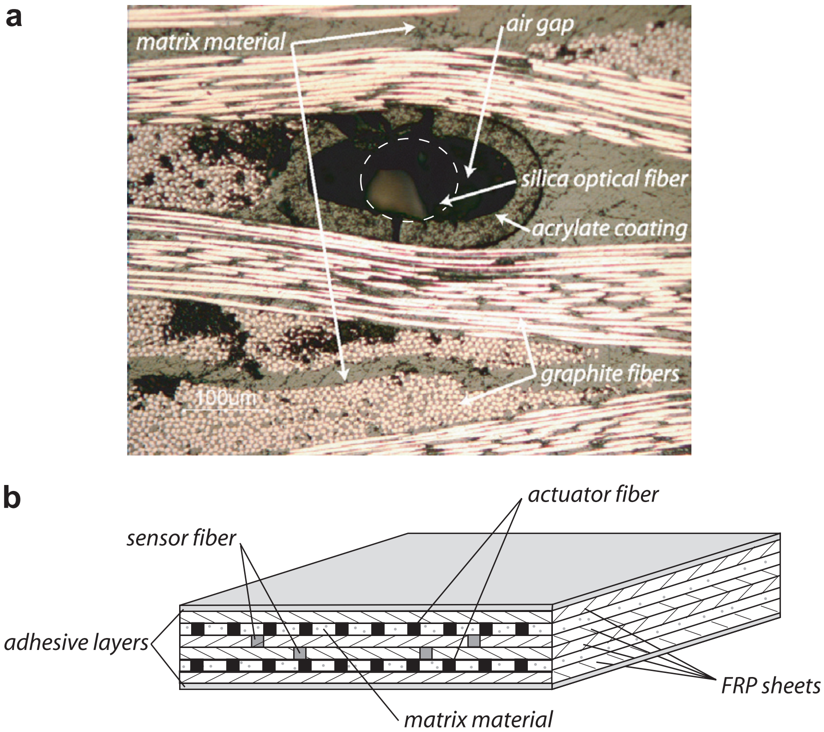

Fig. 1. Example systems for application of optimal 3D shear-lag model: (a) optical micrograph of cross-section of fiber sensor embedded between plies of a woven laminate graphite/epoxy composite system; (b) idealized model of laminated structure with embedded actuators and sensors.

two examples of fiber reinforced composites with embedded sensors and actuators where such model could be applied.

With recent advances in experimental methods such as micro-Raman spectroscopy, it has been possible to evaluate the shear-lag method predictions for load transfer between neighboring fibers. In general, the shear-lag method well predicts the load transfer in high volume polymer matrix composites. However, this is not the case for composites with matrix to fiber modulii ratios near unity due to the increased role of the matrix axial stresses (Beyerlein and Landis, 1999). Stress concentrations in neighboring fibers due to fiber breakage have also been shown to depend on the matrix modulus (Wagner et al., 1996). Anagnostopoulos et al. (2005) induced local fiber discontinuities in high volume fraction aramid/epoxy composites and applied laser Raman spectroscopy to demonstrate that the quality of the interface was maintained, justifying the use of an elastic transfer model. Furthermore, the authors determined that at applied strains below the residual strain threshold in high volume composites, the local residual stresses strongly influence the local stress transfer. This effect can be accounted for in shear-lag analyses, however, the local shear modulus of the "as-processed" composite must be known. Once the residual strains have been overcome by the applied strains the shear-lag method based on the shear modulus of the bulk matrix material works well. These experimental results emphasize the importance of the role of the matrix axial and residual stresses, however, give strong support to the use of shear-lag methods for appropriate applications.

Most of the early shear-lag models were based on the assumption that the matrix material shear behavior is controlled by the axial displacements of the surrounding fibers and the role of its axial stiffness can therefore be neglected. The reduced role of the matrix is therefore merely to transfer axial loads in adjacent fibers through shear deformation, i.e. as a shear spring.[1] As a result, these methods are only suitable for composites

[1] An alternate perspective is that a matrix material with zero axial stiffness yet finite shear stiffness can be interpreted as representing a matrix material that has failed in tension through either cracking or yielding at low stress magnitudes (Landis and McMeeking, 1999).