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Piezo-electric control of stiffened panels subject to interactive buckling

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ABSTRACT

The paper examines the feasibility of piezo-electric control of stiffened plates carrying axial compression and subject to interaction of local and overall buckling. A simple control strategy involving piezo-electric patches along the tips of the stiffeners carrying equal and opposite electric fields to resist bending of the stiffeners was found to effectively counteract the adverse effects of mode interaction and imperfection-sensitivity. For the dynamic problem, this strategy needed to be supplemented with patches attached to the surfaces of the plate in the middle of the panel to damp out local buckling oscillations. Two panels were considered, these being scaled replicas of each other. This enabled an examination of the scaling laws of response with practical applications in view. The results demonstrate that the structural performance of optimally designed stiffened structures can be enhanced with minimal energy consumption by appropriately designed piezo-electric patch configuration.

1. Introduction

Thin-walled structures such as stiffened plates and shells fabricated out of high strength materials are ubiquitous in aerospace structures. These are prone to buckle in a variety of modes with a strong possibility of adverse mode interaction whenever subjected to axial compression and/or bending. Optimally designed stiffened plates subject to compression tend to fail by an interaction of overall and local buckling, and are imperfection-sensitive. This results in an erosion of their projected load carrying capacity based on the critical loads as determined from a linear stability analysis (Thompson and Hunt, 1973; Tvergaard, 1973; Sridharan et al., 1994; Sridharan and Kasagi, 1997). As the axial compression carried becomes a sizable fraction of the critical load, these structures exhibit large amplitude oscillations under disturbances such as lateral acoustic pressure and could experience dynamic instability by divergence. For these structures, the load corresponding to dynamic instability can be significantly less than that which would cause collapse under static conditions (Budiansky, 1965).

Literature on piezo-electric control of flutter, buckling and nonlinear response of plate structures is extensive and we can do no more than mention a few topics investigated in this field: enhancement of column flutter and buckling responses (Wang and Quek, 2002), geometrically nonlinear response of piezo-laminated plates (Rabinovitch, 2005), and active control of nonlinear supersonic panel flutter (Abdel-Motagaly et al., 2005; Li et al., 2007). This paper addresses the issues involved in the piezo-electric control of an "optimally designed" stiffened panel – optimal in the sense the local and overall buckling loads under axial compression are rendered equal. Such structures are known to be imperfection-sensitive due to nonlinear modal interaction. The example chosen for study is however a simple one, a panel consisting of slender plate and relatively stocky stiffener-designated in the literature as Tvergaard panel-1 (Tvergaard, 1973; Sridharan et al., 1994). It is shown that feedback voltages across patches at the stiffener tips, proportional to the bending strain have a salutary effect in stiffening the structure at loads that exceed the capacity of the uncontrolled structure under static conditions. In this case local buckling deflections are allowed to occur, but they are seen to be innocuous in so far as the overall bending has been controlled. Next the feasibility of damping out of large amplitude oscillations liable to be triggered at loads smaller than the dynamic buckling load is studied. As before the control is exercised using piezo-electric actuators attached at the stiffener tips only. The feedback gains are now proportional to the strain-rates sensed at the stiffener tips. This has the effect of damping out overall oscillations fairly quickly, but local mode vibrations tend to linger on for a long duration. In an attempt to damp out the local (plate) vibrations additional control is exercised via piezo-electric actuator patches placed at upper and lower surfaces at the middle of each plate panel. The feedback gains are proportional to the sum of the strain-rates sensed in the longitudinal and transverse directions. This was found to be very effective in damping out the plate vibrations. Thus by selective use of piezo-electric patch actuators at key locations it was possible to maintain the stiffness of the stiffened plate and damp out the oscillations. Finally the control of a panel with scaled up geometry is studied with practical applications in view with encouraging results.

The analysis of the stiffened plate employs an approach in which the interaction is accounted for by embedding the local

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buckling deformation (Sridharan et al., 1994). This approach isolates the local buckling deformation, together with the second order effects, its variation spatially over the panel and the corresponding components of feedback voltage from the overall effects. This affords a greater insight into the response of the stiffened panel than conventional finite element schemes and makes possible a more focused control strategy. These aspects of analysis are reviewed briefly in the following section. Nevertheless, the specific contribution of the paper is in the realm of establishing a viable strategy of piezo-electric control of stiffened panels, with an eye on the practical application.

2. Theory

In this section, the theory and formulation of the present finite element model is outlined. Fig. 1 shows a "wide" stiffened plate and a typical panel consisting of plate elements on either side of a stiffener.

2.1. Displacement, strain and stress vectors

The displacement variables are:

$$\{u\}^T = \{u, v, w, \alpha, \beta\} \tag{1}$$

where u, v and w are the displacement components in the axial (x-), transverse (y-) and outward normal (z-) directions, respectively, at any point on the middle surface plate or stiffener (Fig. 2) and α and β are the rotations of the normal in the xz and yz planes, respectively (Sridharan et al., 1992).

The generic strain vector $\{\varepsilon\}$ may now be defined as in Reissner–Mindlin theory:

$$\{\varepsilon\}^T = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \chi_x, \chi_y, \chi_{xy}, \gamma_{xz}, \gamma_{xz}\} \tag{2}$$

where

$$\{\bar{\varepsilon}\} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}\} \tag{3a}$$

are the in-plane strain components at the plate mid-surface,

$$\{\chi\} = \{\chi_x, \chi_y, \chi_{xy}\} \tag{3b}$$

are the curvature components, and

$$\{\gamma\} = \{\gamma_{xz}, \gamma_{yz}\} \tag{3c}$$

are the transverse shearing strain components.

The generic stress vector $\{\sigma\}$ conjugate with $\{\varepsilon\}$ consist of stress resultants. These consist of the force resultants $\{N\} = \{N_x, N_y, N_{xy}\}$, moment resultants $\{M\} = \{M_x, M_y, M_{xy}\}$ and transverse shear forces $\{Q\} = \{Q_x, Q_y\}$. The generic stress–strain relations are taken in the standard form:

$$\begin{aligned} N_i &= A_{ij}\bar{\varepsilon}_j + B_{ij}\chi_j \\ M_i &= B_{ij}\bar{\varepsilon}_j + D_{ij}\chi_j \end{aligned} \quad (j = 1,2,6) \ (i = 1,2,6) \tag{4a–b}$$

$$Q_i = k\bar{G}t\gamma_i \quad (i = 1,2) \tag{4c}$$

where $[A]$, $[B]$, $[D]$ are well known matrices in the literature on layered composites, $\overline{G}$ is the averaged transverse shear modulus, k is the shear correction factor (=5/6) and t is thickness of the plate element. These equations may be written in the abbreviated form:

$$\sigma_i = H_{ij}\varepsilon_j \tag{5}$$

The following strain–displacement relations are used for the plate structure:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left\{\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right\} \\ \varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ \chi_x &= \frac{\partial \alpha}{\partial x}; \quad \chi_y = \frac{\partial \beta}{\partial y}; \quad \chi_{xy} = \frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x} \\ \gamma_{xz} &= \alpha + \frac{\partial w}{\partial x}; \quad \gamma_{yz} = \beta + \frac{\partial w}{\partial y} \end{aligned} \tag{6a–h}$$

These are but von Karman plate equations modified to account for transverse shear deformation and the large in-plane movements of stiffeners such as occur under overall buckling/bending.

The strain–displacement relations can be expressed in the abbreviated form:

$$\varepsilon_i = L_{1ij}(u_j) + \frac{1}{2}L_{2ij}(u_j) \quad (i = 1, \ldots 8) \ (j = 1, \ldots 5) \tag{7}$$

where L_1 strands for linear differential operators and L_2 for a quadratic operators implicit in Eqs. 6(a–h).

Under axial compression there are two characteristic modes of buckling, viz. the overall buckling associated with a long wave mode and local buckling characterized by a sinusoidal mode with a number of half waves (m). In the former stiffener undergoes significant in-plane displacements whereas in the latter the plate-stiffener junction remains immobile, as the plate buckles between the stiffeners.

2.2. Solution of the local buckling problem

2.2.1. Linear stability analysis

The following notation will be employed in the following. A superscript (1) indicates a first order local buckling quantity (Eigen mode), a superscript (2) indicates a second order quantity and a

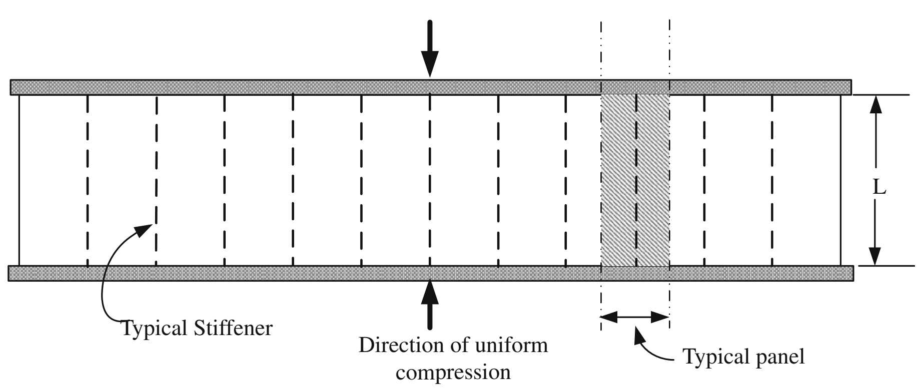

Fig. 1. "Wide" stiffened plate and a typical panel.