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Plastic buckling analysis of thick plates using p-Ritz method

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Abstract

This paper is concerned with the application of the p-Ritz method for the plastic buckling analysis of thick plates. In order to allow for the effect of transverse shear deformation in thick plates, the Mindlin plate theory is adopted. The plastic buckling behaviour of the plate is captured by using the incremental and deformation theories of plasticity. The material property of the plate is assumed to obey the Ramberg–Osgood stress–strain relation. The p-Ritz method will be applied to obtain the governing eigenvalue equation for the plastic buckling analysis of uniformly stressed plates with edges defined by polynomial functions. In the p-Ritz method, the displacement functions of the plate are approximately represented by the product of mathematically complete two-dimensional polynomial functions and boundary equations raised to appropriate powers that ensure the satisfaction of the geometric boundary conditions. The validity, convergence and accuracy of the method were demonstrated for various plate shapes such as rectangular, triangular and elliptical shapes. A parametric study was also undertaken to study the plastic buckling behaviour and the effect of transverse shear deformation.

Keywords: Ritz method; Plastic buckling; Mindlin plate theory; Thick plates

1. Introduction

The Ritz method is a commonly used method for plate analysis because of its simplicity. In this method, it is necessary to approximate the displacement with suitable functions that satisfy the geometric boundary conditions. By minimizing the total potential energy functional with respect to the parameterized displacement functions, one obtains a system of homogeneous equations which forms the governing eigenvalue equation. Since the convergence and accuracy of the method depend on the selection of appropriate displacement functions, many researchers have proposed various admissible functions such as spline functions (Mizusawa et al., 1979), and beam characteristic orthogonal polynomials (Bath, 1985). A breakthrough in automating the Ritz method for analysis of a wide range of plate shapes and boundary conditions may be credited to Liew and Wang (1992, 1993). They proposed that the Ritz functions be defined by the product of mathematically complete two-dimensional polynomial functions and boundary equations raised to appropriate powers that ensure the satisfaction of the geometric boundary conditions. They published a series of papers on the elastic buckling

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analysis of plates using the so-called p-Ritz method (Wang et al., 1993a; Wang et al., 1994; Liew and Wang, 1995). Smith et al. (1999) studied the convergence of simple polynomials as well as orthogonal polynomials (such as Chebyshev, Legendre, Hermite, and Laguerre) as Ritz functions for uniaxial plate buckling analysis. They concluded that the simple polynomial gives distinct advantages over the orthogonal polynomials due to its simplicity in implementation and computational ease.

With regards to plastic buckling of plates, various theories of plasticity have been proposed in the literature. These theories can be categorized mainly under the incremental (flow) theory of plasticity (Handelman and Prager, 1948) and the deformation theory of plasticity (Illyushin, 1947). So far, only a few papers have adopted the Ritz method for the plastic buckling analysis of plates despite extensive studies on their elastic counterparts. Smith et al. (2003) used the Ritz method for the plastic local buckling analysis of steel plates subjected to in-plane axial, bending and shear loading by using the incremental theory of plasticity. Their study is, however, confined to the treatment of rectangular plates and is based on the classical thin plate theory.

Prompted by the lack of rigorous treatment of the subject, we present the p-Ritz method for plastic buckling analysis of arbitrarily shaped thick plates with boundaries defined by polynomial functions. In addition, we also include the effect of transverse deformation in the formulation so that the buckling solutions are not overpredicted when dealing with relatively thick plates. In order to capture the inelastic effect, we have adopted the incremental and deformation theories of plasticity and the Ramberg–Osgood constitutive relation for the plate material (Ramberg and Osgood, 1943). After providing the p-Ritz formulation for the plastic buckling of plates, the validity, convergence and accuracy of the method in computing the plastic buckling loads are demonstrated by considering rectangular, triangular and elliptical plate examples. The plastic buckling behaviour and the effect of transverse shear deformation are also investigated.

2. Problem definition and formulation

Consider an isotropic, arbitrarily shaped plate of uniform thickness h and subjected to a uniform in-plane compressive stress σ as shown in Fig. 1. The plate edges may be defined by polynomial functions. The edges may take on any combination of simply supported, clamped and free edges. The plate material is assumed to obey the Ramberg–Osgood constitutive relation as depicted in Fig. 2. The problem at hand is to determine the plastic buckling stress of such an in-plane loaded plate.

According to the Mindlin plate theory, the admissible velocity field is given by (Mindlin, 1951)

$$\upsilon_x = z\phi_x; \quad \upsilon_y = z\phi_y; \quad \upsilon_z = w \tag{1a–c}$$

where ϕ_x and ϕ_y are the rates of rotations in the y and x directions, respectively, and w is the transverse velocity. The strain rates corresponding to Eq. (1a–c) are given by

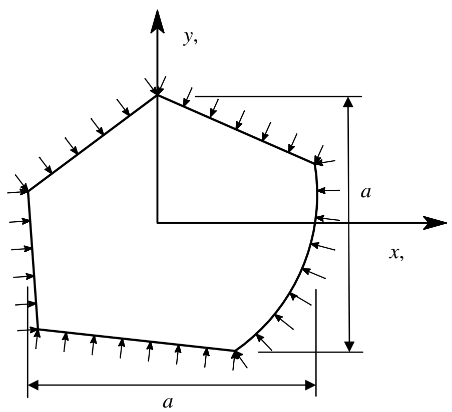

Fig. 1. Mindlin plate of arbitrary shape subjected to uniform in-plane stress σ.